Lesson 24

Orthogonal Complement

6.2

Def Let S be a nonempty subset of an inner product space V. The orthogonal complement of S

$$S^{\perp} = \{ v \in V , \langle v , s \rangle 0 \text{ for all } s \in S \}$$

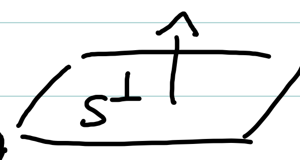
Th1: $S^{\perp} \leq V$

Example $V = R^3$ $S = \{(1,1,2)\}$

$(x,y,z) \perp (1,1,2)$ iff $\langle (x y z),(1,1,2)\rangle = 0$

iff $x+y+2z=0$

$S^\perp = \{(x y z) \mid x+y+2z=0\}$



$= \text{span}\{(1,-1,0) \ (-2,0,1)\}$

Thm: Suppose V is an inner product space and Suppose U is a finite dimensional subspace of V, then $V = U \oplus U^{\perp}$

Proof:

Th 3: If $U \subseteq V$ and V is a finite dimensional vector space then

$\dim U^{\perp} = \dim V - \dim U$

Def: v_1 in the previous proof is called the orthogonal projection of v onto U and denoted by $P_U(v)$

that is given an inner vector space V and a finite dimensional subspace U of V for every $v \in V$ the orthogonal projection of v on U, $P_U(v)$ is the unique vector s.t we can write

$v = P_U(v) + w$ with $P_U(v) \in U$

$w \in U^{\perp}$.

Note:

$$P_U(v) = \langle v\ e_1 \rangle e_1 + \langle v\ e_2 \rangle e_2 + \cdots + \langle v\ e_m \rangle e_m$$

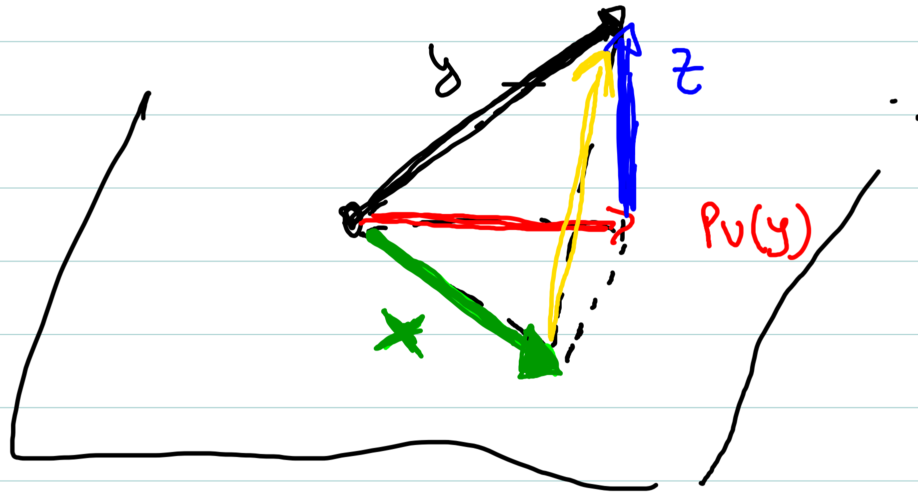
where $e_1\ e_2 \dots e_m$ is an orthonormal basis for U. ($P_U(v)$ does not depend on the choice of the basis)

Th 4: $P_U \quad V \longrightarrow U$

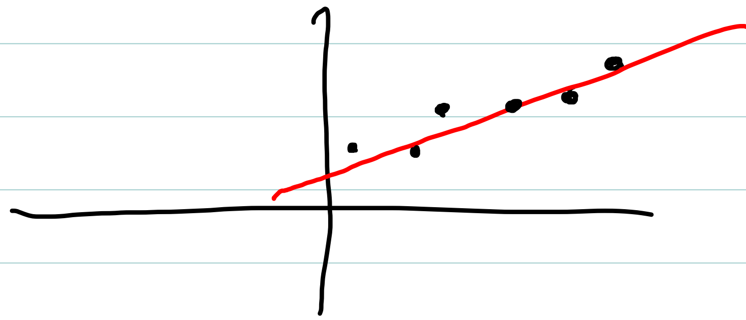
is a linear transformation and $P_U^2 = P_U$

Proof:

Th 5: Let V be an inner product space, and let U be a finite dimensional subspace of V then $\forall y \in V$ $\forall x \in U$ $\|y - x\| \geq \|y - P_U(y)\|$


y
z
Pv(y)
x

Least square approximation data fitting



Goal: find line that best fits

data points (x_1, y_1) (x_2, y_2) ---- (x_n, y_n)

That is find line $y = cx + d$ such that

the error. $$E = \sum_{i=1}^{n} (y_i - cx_i - d)^2$$

is as small as possible.