# Lesson 2

## Subspaces

1.3

Def : A subset $S$ of a vector space $V$ is a subspace of $V$ (sometimes I will write $S \leq V$) if

1) $0 \in S$
2) $u, v \in S \Rightarrow u + v \in S$ for all $u, v$ in $S$
3) $u \in S$, $\lambda \in F \Rightarrow \lambda u \in S$ for all $\lambda \in F$
$u \in S$

So a subspace $S$ of $V$ is itself a vector space with the same addition and scaler multiplication as $V$.

Examples of subspaces:

Ex 1: $V = R^2$

$S = \{(x, y) \mid x + y = 0\}$

Check

Ex 2 : $T = \{(x,y) \mid x+y = 1\}$

Describe all subspaces of $R^2$

Ex : $S = \{\{x_n\} \mid \lim_{n \to +\infty} x_n = 0\} \leq \mathbb{R}^\infty$

1) The 0 sequence i.e $x_n = 0$ for all $n$ is in $S$

2) If $\{x_n\}, \{y_n\} \in S$ we need to show $\{x_n\} + \{y_n\} = \{x_n + y_n\} \in S$ :

If $\lim_{n \to +\infty} x_n = 0$ and $\lim_{n \to +\infty} y_n = 0$

then $\lim_{n \to +\infty} (x_n + y_n) = 0$ .

so $\{x_n\} + \{y_n\} = \{x_n + y_n\} \in S$

3) If $\{x_n\} \in S$ then $\lim_{n \to +\infty} x_n = 0$ so $\forall \alpha \in \mathbb{R}$

$\lim_{n \to +\infty} \alpha x_n = 0$ so $\alpha \{x_n\} = \{\alpha x_n\} \in S$

Ex What about $S = \{\{x_n\} \mid x_n \in \mathbb{R}, \lim_{n \to +\infty} x_n = 1\}$ ?

$0 \notin S$ so not a vector space

Example:

$$S = \{ (x, y) \mid x \in \mathbb{Z},\ y \in \mathbb{Z} \} \subseteq \mathbb{R}^2$$

Ex $S = \{(x\ 0) \mid x \in \mathbb{R}\} \cup \{(0\ y)\ y \in \mathbb{R}\}$
$\subseteq \mathbb{R}^2$

## Set operations and subspaces.

Th Let $S \leq V$, $T \leq V$ then
$S \cap T \leq V$

Proof :

Let $V = R^2$

$S = \{ (x, 0) \mid x \in R \}$

$T = \{ (0, y) \mid y \in R \}$

$S \cup T$ is not a subspace of $R^2$.