Lesson 18

Eigenspaces E_λ

diagonalizability

5.2

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A is $n \times n$	$T \quad V \to V$

λ is eigenvalue

$v \neq 0$ eigenvector

$Av = \lambda v$	$Tv = \lambda v$
$(A - \lambda I)v = 0$	$(T - \lambda I)v = 0$
$v \in N(A - \lambda I)$	$v \in N(T - \lambda I)$
$A - \lambda I$ not invertible	$T - \lambda I$ not invertible
$\det(A - \lambda I) = 0$	

Eigenvalues and eigenvectors of A are those of $L_A \; F^n \to F^n$ $L_A v = A \cdot v$	If $\dim V < \infty$ $B = \{v_1, \dots v_n\}$ is a basis for V eigenvalues of T are those of T_B^B

and if $(x_1, \dots x_n)$ is an eigenvector for T_B^B then $x_1 v_1 + \cdots + x_n v_n$ is an eigenvector for T

Def Suppose $T \in \mathcal{L}(V)$ and λ is an eigenvalue for T then

$$N(T-\lambda I) = \{v \mid Tv = \lambda v\}$$

is called the eigenspace of λ and it is denoted by $E(\lambda, T)$ or $E_\lambda(T)$ or E_λ

It is the set of all eigenvectors of T with eigenvalue λ, plus the 0 vector

Th 1: If $T \in \mathcal{L}(V)$ and λ is an eigenvalue of T, $E_\lambda \leq V$ and E_λ is T invariant.

Th 2 : Suppose that $\lambda_1, \dots \lambda_m$ are distinct eigenvalues of T and $v_1, \dots v_m$ are corresponding eigenvectors. Then $v_1 \cdots v_m$ are linearly independent.

Th 3 If $T\ V \to V$ is a linear transformation and $\lambda_1, \dots \lambda_n$ are distinct eigenvalues for T the sum $E_{\lambda_1} + \dots + E_{\lambda_n}$ is direct

Th4: Let V be finite dimensional and $T \in \mathcal{L}(V)$; let $\lambda_1, \lambda_2 \cdots \lambda_m$ be the distinct eigenvalues of T. Then the following are equivalent

1) T is diagonalizable
2) V has a basis consisting of eigenvectors of T
3) $V = E_{\lambda_1} \oplus \cdots \oplus E_{\lambda_m}$
4) $\dim V = \dim E_{\lambda_1} + \cdots + \dim E_{\lambda_m}$

Def $C^\infty(\mathbb{R})$: functions f $\mathbb{R} \to \mathbb{R}$

s.t $f^{(j)}$ exists for all j

Example

$T \; C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ (Not finite dimensional)

$T(f) = f'$

f is an eigenvector for T with eigenvalue λ if

$T(f) = \lambda f$ That is if f satisfies the differential equation $f' = \lambda f$.

The solutions of this differential equation are functions $f(x) = c\, e^{\lambda x}$

So every λ in $\mathbb{R}$ is an eigenvalue and the associated eigenvectors are functions $c\, e^{\lambda x}$ $c \neq 0$.

If $\lambda = 0$ the eigenvectors are non zero constant functions.