Winter 2023 Math 318 midterm

NAME (First,Last) : ....

Student ID ....

UW email ....

- Please write your name as it appears in the Canvas roster.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** justify your answers and show your work.
- Your work needs to be neat and legible.

Problem 1. Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

1. The linear transformation $T : R^2 \to R^2$, $T(v) = Av$ is a **clockwise** rotation of an angle θ: what is θ?

2. Does A have any real eigenvalues ? Justify your answer.

3. Argue that if α is an eigenvalue for a matrix B, then α^2 is an eigenvalue for B^2.

4. If we consider only real (not complex) eigenvalues, is it true that if B^2 has real eigenvalue μ , then B has real eigenvalue $\alpha = \sqrt{\mu}$? Explain.

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Problem 2. A 1970 survey determined that 50% of American car owners drove large cars, 30% drove middle sized cars and 20 % drove small cars. A second survey in 1980 determined that 40% of large car owners in 1970, still drove a large car in 1980, but 10% switched to a middle sized car and 50% switched to a small car; of the middle sized car owners in 1970, 20% had switched to a big car, 70% were still driving a middle sized car, and 10% were owning a small car;finally, of the small car owners in 1970, 20% owned a large car in 1980, 20% owned a middle sized car, and 60 % still owned small cars.

1. Use a Markov matrix M to describe this situation, that is you want to find a matrix M, such that $M \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ where a, b, c represent the percentages of car owners driving large, medium sized and small cars respectively in 1980.

2. $\begin{bmatrix} 0.5 \\ 0.7 \\ 0.8 \end{bmatrix}$ is an eigenvector for M. What is the eigenvalue associated with it ? Explain how you know.

3. Assuming that every 10 years American switch cars as described above, determine the percentage of Americans that will own large, medium sized and small cars respectively in the long run. Show your work and explain how you got your answer.

Problem 3.

1. In a hw problem you proved that if P is the matrix of a projection, then P is diagonalizable and the only eigenvalues of P are λ =0 or 1. Prove that the converse is true, that is if A is a diagonalizable matrix whose only eigenvalues are 0 or 1, then A is the matrix of a projection, that is $A^2 = A$.

2. Give an example of a matrix A having only real eigenvalues, and no eigenvalues different from 0 or 1, which is not the matrix of a projection. You must explain why your example works.

3. Let W be span ($\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$). Calculate the orthogonal projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ on W.Show your work and explain how you got your answer.

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(Problem 3 continued)

4. Let V be the plane x+y+z=0 . Calculate the orthogonal projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ on V.Show your work and explain how you got your answer.