Which sets are denumerable? Infinite sets that look like Z^+ i.e elements can be listed: first, second, third, ...

Denumerable like Z^+ smallest infinite sets } non denumerable lots of sets $\mathbb{R}$ $[0,1]$ $P(Z^+)$ lots of other stuff

Worksheet4

1. Prove that if you have denumerably many many disjoint sets A_n , all of which are denumerable, their union $\cup_n A_n = \{x|x \in A_n$ for some $x \in Z^+\}$ is also denumerable.
2. Suppose A is a set. Prove that $|A| < |P(A)|$. You can assume A is infinite.

This means

1) There is f $A \longrightarrow P(A)$ f injective

2) No f $A \longrightarrow P(A)$ can be surjective.

To prove an infinite set A is denumereble:

1) Show $A \subseteq B$ with B denumereble

or

2) Find a bijection f $A \to B$ or f $B \to A$ with B denumereble

or

3) Show $A = A_1 \cup A_2$ with A_1, A_2 denumereble

To prove an infinite set A is not denumereble

1) Use Cantor's diagonalization argument to prove any f $\mathbb{Z}^+ \to A$ cannot be surjective

2) Find a bijection f $A \to B$ or f $B \to A$ with B non denumereble