Worksheet3. The binomial coefficient. Sets

1. (From last week) Prove that the number of binary strings of length n that do not contain two consecutive 1 is u_{n+2} , the n+2 $^{\text{nd}}$ Fibonacci number.

2. Is $P(A \cup B) = P(A) \cup P(B)$? Prove your answer.

3. Consider the double sequence $\{a_{nm}\}$ defined as follows:

   $a_{n0} = 1$ for all $n \geq 0$

   $a_{0m} = 0$ for all $m \geq 1$

   $a_{n+1\,m+1} = a_{nm} + a_{n\,m+1}$ for $n \geq 0, m \geq 0$

   (a) Draw a table showing the values of a_{nm} for $0 \leq n, m \leq 5$.

   (b) When $n \geq m$, a_{nm} is also denoted $\binom{n}{m}$. Prove that if $n \geq m$

   $$a_{nm} = \frac{n!}{m!\,(n-m)!}$$

4. Prove that $A \times B = B \times A \Rightarrow A = B$