

Lesson 9 .

Sets

A set is a collection of objects.

$$\text{Ex: } A = \{1, 2, 3\} \quad 1 \in A, 5 \notin A$$

$$\begin{aligned} \text{Ex: } B &= \{n \in \mathbb{Z}^+ \mid n \text{ is even}\} = \\ &= \{2, 4, 6, 8, \dots\} \end{aligned}$$

$$\begin{aligned} \text{Ex: } C &= \{n^2 \mid n \in \mathbb{Z}^+\} = \\ &= \{1, 4, 9, 16, \dots\} \end{aligned}$$

The order of the elements of a set does not matter: $\{1, 2, 3\} = \{3, 2, 1\}$

A set has no repeated elements

$$A = \{\{1\}\} \text{ is a set } \{1\} \in A, 1 \notin A$$

\emptyset or $\{\}$ is the empty set

Suppose $A = \{1, 2, 3\}$ $B = \{A\}$

$$C = \{x \in A \mid x \geq 2\}$$

Is $1 \in A$? Y

Is $1 \in B$? N

Is $1 \in C$? N

Is $1 \in \phi$? N

Def: $A \subseteq B$ (A is a subset of B)
iff $\forall x \quad x \in A \Rightarrow x \in B$

Ex: $A = \{1, 2, 3\}$ $B = \{1, 2\}$
 $C = \{0, 1\}$ $D = \{A\}$

Is $B \subseteq A$? Y

Is $A \subseteq B$? N

Is $C \subseteq A$? N

Is $D \subseteq A$? N

Is $A \subseteq D$? N

Is $\phi \subseteq A$? Y

Is $A \subseteq A$? Y

Two sets are equal if they have the same elements.

$$\text{Note } A = B \Leftrightarrow ((A \subseteq B) \wedge (B \subseteq A))$$

Set operations :

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$\text{Ex } \{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$$

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$\text{Ex } \{1, 2\} \cap \{2, 3\} = \{2\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$\text{Ex : } \{1, 2\} - \{2, 3\} = \{1\}$$

If there is an Universe U

$$A^c = U - A \quad (\text{the complement of } A)$$

Suppose $U = \mathbb{Z}$, $EVEN = \{n \in \mathbb{Z} \mid n \text{ is even}\}$
 $ODD = \{n \in \mathbb{Z} \mid n \text{ is odd}\}$

$$EVEN \cup ODD = \mathbb{Z}$$

$$EVEN \cap ODD = \emptyset$$

$$EVEN - ODD = EVEN$$

$$EVEN^c = ODD$$

Def: If A and B are sets,
 A and B are disjoint means
 $A \cap B = \emptyset$.