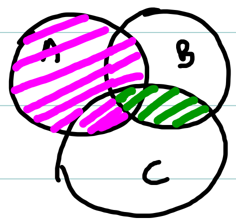
Lesson 10

Sets

Th: $\underbrace{A \cup (B \cap C)}_{S_1} = \underbrace{(A \cup B) \cap (A \cup C)}_{S_2}$



Not a proof, why?

Proof: First we shall prove $S_1 \subseteq S_2$:
suppose $x \in S_1$ then $x \in A$ or $x \in B \cap C$.
If $x \in A$ then $x \in A \cup B$ and $x \in A \cup C$,
therefore $x \in (A \cup B) \cap (A \cup C)$.
If $x \in B \cap C$ then $x \in B$ and $x \in C$, so
$x \in A \cup B$ and $x \in A \cup C$, therefore $x \in (A \cup B) \cap (A \cup C)$.
Then we shall prove $S_2 \subseteq S_1$: suppose $x \in S_2$, then
$x \in A \cup B$ and $x \in A \cup C$. If $x \in A$ then
$x \in A \cup (B \cap C)$. If $x \notin A$ then $x \in B$ and C
so $x \in B \cap C$ and therefore $x \in A \cup (B \cap C)$.

Truth tables proof:

$x \in$ A	B	C	$A \cup (B \cap C)$	$(A \cup B) \cap (A \cup C)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F
F	T	F	F	F
T	F	F	T	T
F	F	F	F	F

The last 2 columns are equal.

Some useful properties

1) Associativity of $\cup$ and $\cap$:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

2) Commutativity of $\cup$ and $\cap$:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

3) Distributivity :

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4) De Morgan laws :

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

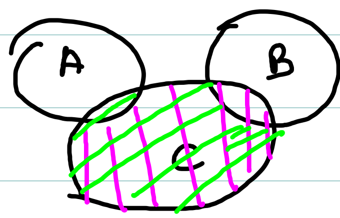
Th $\underbrace{(A \cup B)^c}_{S_1} = \underbrace{A^c \cap B^c}_{S_2}$

Proof:

A	B	A^c	B^c	$A \cup B$	$(A \cup B)^c$	$A^c \cap B^c$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Note that the last two columns are the same.

Th: $A \cap B = \phi \Rightarrow C = \underbrace{(C-A) \cup (C-B)}_{S_1}$



Proof: assume $A \cap B = \phi$

First we shall prove $C \subseteq S_1$:

assume $x \in C$; if $x \notin A$ then $x \in C - A$ so $x \in S_1$; if $x \in A$ then $x \notin B$ (since $A \cap B = \phi$) and therefore $x \in C - B$ so $x \in S_1$

Now we will show $S_1 = (C-A) \cup (C-B) \subseteq C$

Assume $x \in S_1$ then $x \in C-A$ or $x \in C-B$ but in any case $x \in C$.

Consider the truth table

A	B	C	C-A	C-B	(C-A)∪(C-B)
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	T	T	T	T
F	T	F	F	F	F
T	F	F	F	F	F
F	F	F	F	F	F

Why isn't the third column equal to the sixth?

Suppose that $A_1, A_2, \ldots A_n \ldots$ are sets then:

Def $\bigcup_{i=1}^{n} A_i = \{x \mid x \in A_i \text{ for some } i,\ 1 \le i \le n\}$

$\bigcap_{i=1}^{n} A_i = \{x \mid x \in A_i \text{ for all } i\ \ 1 \le i \le n\}$

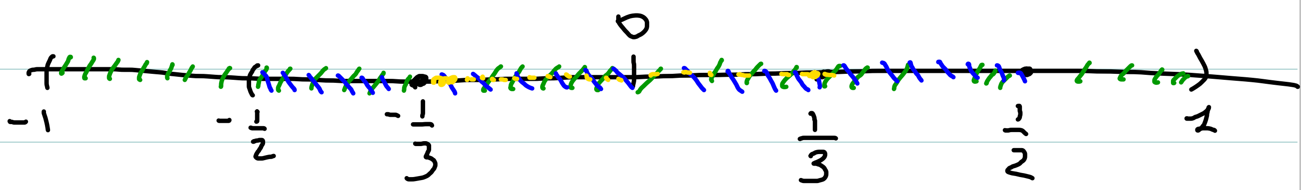
$\bigcup_{i=1}^{\infty} A_i = \{x \mid x \in A_i \text{ for some } i \in \mathbb{Z}^+\}$

$\bigcap_{i=1}^{\infty} A_i = \{x \mid x \in A_i \text{ for all } i \in \mathbb{Z}^+\}$

Ex $A_i = \left(-\frac{1}{i}, \frac{1}{i}\right)$

$\bigcup_{i=1}^{\infty} A_i = ?$

$\bigcap_{i=1}^{\infty} A_i = ?$



$$\bigcup_{c=1}^{\infty} A_c = (-1, 1)$$

$$\bigcap_{c=1}^{\infty} A_c = \{0\}$$