Lesson 1

What is a proof ?
The language of math
Connectives: $\wedge \ \vee \ \neg \ \Rightarrow$

What is a proof?

① A piece of writing meant for an audience

② It provides a "rigorous argument" that a statement is True.

The formal way

First establish Axioms and rules of inference
A proof of statement A is a finite list of statements that ends in B
Eeach statement in the list is either an axiom or it is derived from statements that precede it in the list, by using inference rules.

In this class "axioms" (i.e what you can assume without proving it) are algebre facts.

What is a statement ?

A (mathematically meaningful) sentence that is unequivocally true or false.

Math is fun is not a statement

$x > 2$ is not a statement.

Every even integer greater than 2 is the sum of two prime numbers is a statement (although at the moment nobody can tell for sure wether it is true or false)

Propositional logic :

Proposition is a statement that is unequivocally true or false

Proposition can be combined using connectives to form more complicated propositions.

Connectives: $\neg$ (not), $\wedge$ (and), $\vee$ (or)
$\Rightarrow$ (implies) $\Leftrightarrow$ (if and only if)

When looking at a proposition we only care whether it is true or false
Atomic (basic) propositions are obviously true or false

Ex : $2 > 1$, $0 = 1$

To say wether $(2>1) \wedge (0=1)$ is True or false, we need to explain what " $\wedge$ " means.

Propositional variables: P, Q, R ... stand for (arbitrary) propositions

Propositional formulas: are built using propositional variables and connectives.

Ex $P \vee (\neg Q \Rightarrow (P \wedge R))$

To define the meaning of connectives we build Truth tables

P	$\neg P$
T	F
F	T

Unary connective

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex $(2 > 1) \vee (3$ is prime$)$ is T

A predicate (open statement) contains free variables (not bound by a quantifier)

Quantifiers: $\forall$ (for all) $\exists$ (exists)

We will talk about quantifiers later in the course.

Ex $x > 2$ is a predicate

$\forall x$ in $\mathbb{Z}$ $x > 2$ is a statement. (but not a propositional statement)

Is

$\forall x$ in $\mathbb{Z}$ $((x > 2) \wedge (x$ is prime$))$ => x is odd

True or false?

True. This means it is true when

$x = 3$: $((3 > 2) \wedge (3 \text{ is prime})) \Rightarrow 3 \text{ is odd}$ must be true

$x = 4$: $((4 > 2) \wedge (4 \text{ is prime})) \Rightarrow 4 \text{ is odd}$ must be true

$x = 9$: $((9 > 2) \wedge (9 \text{ is prime}) \Rightarrow 9 \text{ is odd})$ must be true

Truth table for $\Rightarrow$

P	Q	P => Q
T	T	T
T	F	F
F	T	T
F	F	T