Math 300 A Spring 2023 Midterm

NAME (First,Last) : ....

Student ID ....

UW email ....

- Please write your name exactly as it appears in the Canvas 's roster.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Do not turn in any scratch paper.
- Unless stated otherwise, you **MUST** justify your answers and explain why your examples work.
- Your work needs to be neat and legible.

Problem 1 Is $(P \Rightarrow Q) \Rightarrow Q$ equivalent to $P \Rightarrow (Q \Rightarrow Q)$? Justify your answer.

P	Q	P => Q	Q => Q	(P => Q) => Q	P => (Q => Q)
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

No, when P and Q are both false
(P => Q) => Q is false, but P => (Q => Q) is true

NAME (First Last):

Problem 2 Prove or disprove (i.e prove it is false) the following statements. P(A) is the power set of A.

1. $\exists x \in P(P(Z)), \forall y \in P(Z), \quad y \in x$

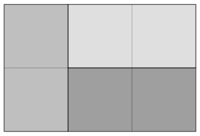
True take $X = P(Z)$ then for any $y \in P(Z)$
$y \in X$

$x \neq \phi \Rightarrow$

2. $\forall x \in P(Z), \exists y \in Z \quad \{y\} \subseteq x$

True : given $X \neq \phi$ in $P(Z)$ then X must contain
some element $y \in Z$ therefore $\{y\} \subseteq x$

Problem 3 Consider a 2xn board. Prove that there are u_{n+1} (the $(n+1)^{\text{st}}$ Fibonacci number) different ways to tile it using 1x2 tiles. Different means that at least one square in the board is covered by a tile placed vertically in one tiling and by a tile placed horizontally in the other. Below is an example of one possible tiling of a 2x3 board.



Recall that the Fibonacci numbers are defined by:

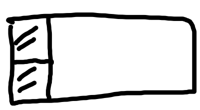
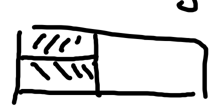
$$u_1 = 1,\ u_2 = 1,\ u_{n+1} = u_n + u_{n-1}$$

Proof by induction:

Base cases

If $n=1$ a 2×1 board can be tiled in 1 way and $U_2 = 1$.

If $n=2$ a 2×2 board can be tiled in 2 ways: 2 horizontal tiles or 2 vertical tiles and $U_3 = 2$

Inductive step: assume $n \geq 2$ and a $2\times n$ board can be tiled in U_{n+1} ways and a $2\times(n-1)$ board can be tiled in U_n ways and consider a $2\times(n+1)$ board. We can tile it, starting from the left, by first placing a vertical tile  then tiling the remaining $2\times n$ board in U_{n+1} possible ways, or first placing 2 horizontal tiles  and then tiling the remaing $2\times(n-1)$ board in U_n ways; so we have a total of

$U_n + U_{n+1} = U_{n+2}$ ways.

NAME (First Last):

Problem 4 Give an example of a function $f : Z^+ \to Z^+$ that is surjective but not injective. You need to both define f and prove that it is surjective and not injective.

$f: \quad Z^+ \longrightarrow Z^+$

$$f(n) = \begin{cases} 1 & \text{if } n = 1 \\ n-1 & \text{if } n > 1 \end{cases}$$

Not injective: $f(1) = 1 = f(2)$

Surjective: given $y \in Z^+$ take $n = y+1$ then $n > 1$ and $f(n) = n-1 = (y+1)-1 = y$