Math 300 A Spring 2023 Midterm

NAME (First,Last) : .....

Student ID .....

UW email .....

- Please write your name exactly as it appears in the Canvas 's roster.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of every odd page of this exam.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Do not turn in any scratch paper.
- Unless stated otherwise, you **MUST** justify your answers and explain why your examples work.
- Your work needs to be neat and legible.

Problem 1 Is $(P \Rightarrow Q) \Rightarrow Q$ equivalent to $P \Rightarrow (Q \Rightarrow Q)$? Justify your answer.

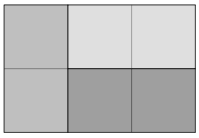
NAME (First Last):

Problem 2 Prove or disprove (i.e prove it is false) the following statements. P(A) is the power set of A. $\emptyset$ is the empty set.

1. $\exists x \in P(P(Z)), \forall y \in P(Z), \quad y \in x$

2. $\forall x \in P(Z), (x \neq \emptyset \Rightarrow \exists y \in Z \quad \{y\} \subseteq x)$

Problem 3 Consider a 2xn board. Prove that there are u_{n+1} (the (n+1)$^{\text{st}}$ Fibonacci number) different ways to tile it using 1x2 tiles. Different means that at least one square in the board is covered by a tile placed vertically in one tiling and by a tile placed horizontally in the other. Below is an example of one possible tiling of a 2x3 board.



Recall that the Fibonacci numbers are defined by:

$$u_1 = 1,\ u_2 = 1,\ u_{n+1} = u_n + u_{n-1}$$

NAME (First Last):

Problem 4 Give an example of a function $f : Z^+ \to Z^+$ that is surjective but not injective. You need to both define f and prove that it is surjective and not injective.