Math 300 A Spring 2023 Final (June 5)

NAME (First,Last) : ....

Student ID ....

UW email ....

- Please write your name as it appears in the Canvas 's roster.

- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.

- Write your NAME (first, last) on top of every odd page page of this exam.

- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.

- Do not turn in any scratch paper.

- Unless stated otherwise, you **MUST** justify your answers and explain why your examples work.

- Your work needs to be neat and legible.

- You can use any result proved in class. You can use the fact that $Q, Z^+ \times Z^+$ the union of any finite number n of denumerable sets, and the union of denumerably many denumerable sets are denumerable.

Problem 1 Prove that an 8x8 checkerboard with the square in position (1,1) (top row and leftmost column) removed cannot be covered by 1x3 tiles.

Problem 2 Let A be the subset of the interval $[0, 1]$, containing all real numbers of the form $0.x_1 x_2 \cdots x_n \cdots$ (infinite decimal expansion), where each decimal digit x_i is either 4 or 7. Is A denumerable ? Prove your answer.

Problem 4 Prove that for all nonempty sets A,B and C,

$$(C \times C) - (A \times B) = ((C - A) \times C) \cup (C \times (C - B))$$

Problem 4 Find integers a, b, c and m such that $ac \equiv bc \bmod m$ but $a \not\equiv b \bmod m$. Explain why your example works.

Problem 5 Consider the function $f : Z_{13} \to Z_{13}$ defined by $f(x) = x^2$. Is f injective? Prove your answer.

Consider the function $f : Z_{23} \to Z_{23}$ defined by $f(x) = 2x + 3$. Is f surjective ? Prove your answer.

Problem 6 Prove that $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$