Tangent line to circle C $(x - x0)^2 + (y - y0)^2 = r^2$ through point P (a, b)
Call O $(x0, y0)$ the center of the circle.

CASE 1

P is on the circle :

- Find the slope m of the line OP. $m = \frac{b-y0}{a-x0}$
- Write the equation of the line through P perpendicular to OP:

$$y = b + \frac{-1}{m}(x - a)$$

  This is the tangent line

CASE 2

P is not on the circle:

- We want to find the point of tangency R (c, d). Now c and d are unknowns.
- Write the slope m of the tangent line in two different ways:
  1. $m = \frac{d-b}{c-a}$ because the tangent line is the line PR
  2. $m = -\frac{c-x0}{d-yo}$ because the tangent line is perpendicular to the line OR, which has slope $\frac{d-yo}{c-x0}$.
- Solve the system

  $\frac{d-b}{c-a} = -\frac{c-x0}{d-yo}$ (sets the two expressions for m in 1) and 2) equal to each other)

  $(c - x0)^2 + (d - y0)^2 = r^2$ (says that R is on the circle C)
  (here your variables are c and d)
- For every solution $(c1, d1)$ that you find (you should find two solutions), write the corresponding tangent line $y = d1 + \frac{b-d1}{a-c1}(x - c1)$

Tangent line to circle C $(x - x0)^2 + (y - y0)^2 = r^2$ parallel to line y=mx+b.

The equation of the tangent line is y=mx+c, here m is given c is an unknown. To find c we can look at the system

$(x - x0)^2 + (y - y0)^2 = r^2$
y= mx+c

that finds the intersections of the line and the circle and impose the condition that there is only one intersection. The system symplifies to:

$(x - x0)^2 + (mx + c - y0)^2 = r^2$
y= mx+c

the first equation is a quadratic in x, it has only one solution when $\Delta = 0$ (recall that for a quadratic equation $ax^2 + bx + c$, Δ is $\sqrt{b^2 - 4ac}$. Solving for $\Delta = 0$ gives you the value of c, therefore the tangent line. The only solution to the system for this value of c gives you the point of tangency Q.