Math 546

Spring 2019

Homework 8

Read Chapter 20. Do Problem 20–7, together with the following (required) Supplementary Exercises. (Please note: You need very little from Chapter 20 to do these problems. Problem 20–7 only requires basic properties of the exponential map, and the Supplementary Exercises are continuations of what I did in lecture and don't really require anything more from Chapter 20.)

Supplementary Exercises:

1. Suppose that G and H are Lie groups, that U is a connected neighborhood of the identity in G, and that $\phi : U \to H$ is smooth with the property that $\phi(ab) = \phi(a)\phi(b)$ whenever a, b, and ab are all in U. Now consider triples (c, ψ, V), where $c \in G$, V is a neighborhood of c in G with $V \cdot V^{-1} \subset U$, and $\psi : V \to H$ is smooth with the property that $\psi(a)\psi(b)^{-1} = \phi(ab^{-1})$ whenever a and b are in V. Two such triples (c_1, ψ_1, V_1) and (c_2, ψ_2, V_2) are equivalent if $c_1 = c_2$ and $\psi_1 = \psi_2$ in a neighborhood of c_1. Let $\tilde{G}$ denote the collection of equivalence classes, and define $\pi : \tilde{G} \to G$ by $\pi(c, \psi, V) = c$. Prove that $\tilde{G}$ has a natural topology so that π is a generalized covering map. Use this to prove that if G is simply connected, then ϕ extends to a smooth homomorphism from G to H.

2. Suppose that G and H are Lie groups and that U is a connected neighborhood of the identity in G. Let $\phi : U \to H$ and $\psi : U \to H$ be smooth maps with the property that $\phi(ab) = \phi(a)\phi(b)$ and $\psi(ab) = \psi(a)\psi(b)$ whenever a, b, and ab are all in U. Prove that, if ϕ and ψ induce the same Lie algebra homomorphism, then $\phi = \psi$.

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