NAME Solutions TA'S NAME __________

STUDENT ID __________ SECTION __________

Math 124K
Autumn 2012

Midterm 1
October 23, 2012

Point totals are indicated in parentheses. You must show your work to receive credit. You do not need a calculator for any of the problems; consequently, you will not receive credit for any solution based on calculator computations.

(18) 1. Evaluate the following limits:

a. $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

b. $\lim_{t\to -\pi/2} \frac{\sin t + \sqrt{\sin^2 t + 2\cos^2 t}}{2\cos^2 t}$

c. $\lim_{h\to\infty} \frac{\frac{1}{3h^2+1} - \frac{1}{h^2}}{\frac{1}{h^2} - \frac{1}{h^3}}$

a. $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} -\frac{1}{2x} = -\frac{1}{4}.$

b. $\lim_{t\to -\frac{\pi}{2}} \frac{\sin t + \sqrt{\sin^2 t + 2\cos^2 t}}{2\cos^2 t} = \lim_{t\to -\frac{\pi}{2}} \frac{(\sin t + \sqrt{\sin^2 t + 2\cos^2 t})(\sin t - \sqrt{\sin^2 t + 2\cos^2 t})}{2\cos^2 t\,(\sin t - \sqrt{\sin^2 t + 2\cos^2 t})}$

$= \lim_{t\to -\frac{\pi}{2}} \frac{-2\cos^2 t}{2\cos^2 t\,(\sin t - \sqrt{\sin^2 t + 2\cos^2 t})}$

$= \lim_{t\to -\frac{\pi}{2}} \frac{-1}{\sin t - \sqrt{\sin^2 t + 2\cos^2 t}} = \frac{-1}{-2} = \frac{1}{2}.$

c. $\lim_{h\to\infty} \frac{\frac{1}{3h^2+1} - \frac{1}{h^2}}{\frac{1}{h^2} - \frac{1}{h^3}} = \lim_{h\to\infty} \frac{\frac{1}{h^2}\left(\frac{1}{3+\frac{1}{h^2}} - 1\right)}{\frac{1}{h^2}\left(1-\frac{1}{h}\right)}$

$= \lim_{h\to\infty} \frac{\frac{1}{3+\frac{1}{h^2}} - 1}{1-\frac{1}{h}} = \frac{\frac{1}{3}-1}{1} = -\frac{2}{3}$

(8) 2. The only information known about two functions f and g is that $f(0) = 4 = g(0)$ and that $f'(0) = -1$, $g'(0) = 3$. Using just this information about f and g, compute the following limits.

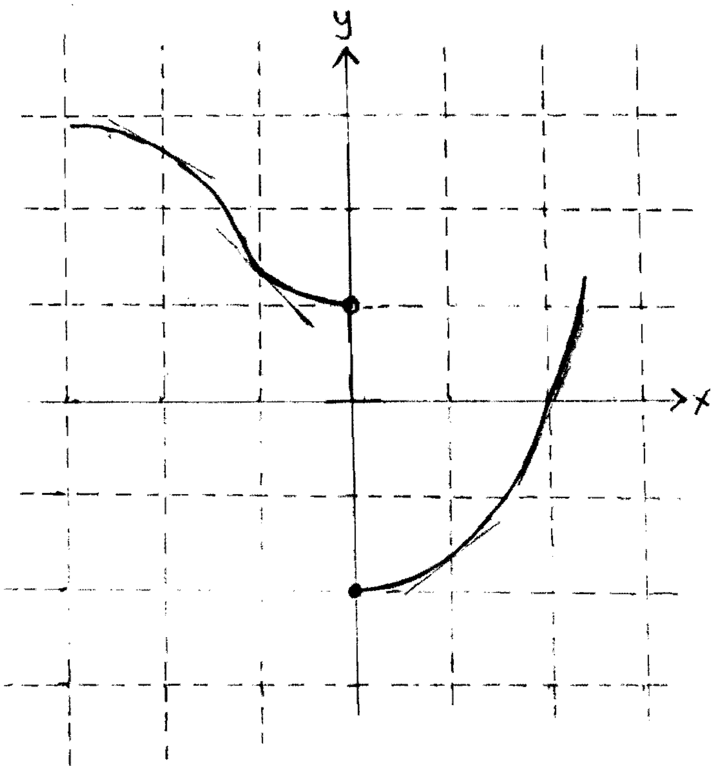
a. $\lim_{h\to 0} \frac{f(h)g(h) - 16}{h}$

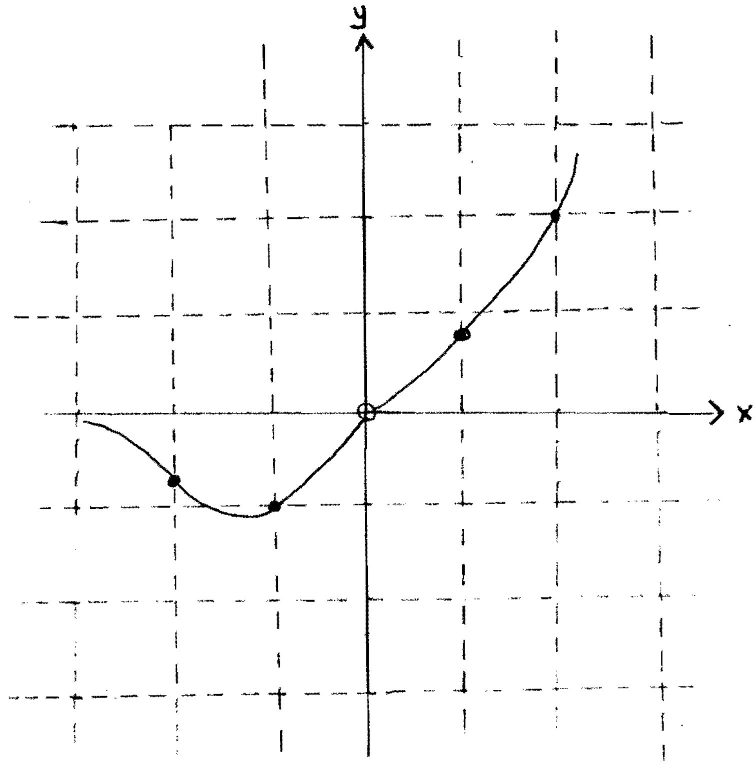
b. $\lim_{h\to 0} \frac{2h(f(h) - 4)}{(g(h) - 4)^2}$

a.
$$\lim_{h\to 0} \frac{f(h)g(h)-16}{h} = \lim_{h\to 0} \frac{f(h)g(h)-f(0)g(0)}{h}$$
$$= (fg)'(0) = f(0)g'(0) + f'(0)g(0)$$
$$= 4\cdot 3 - 1\cdot 4 = 8$$

b.
$$\lim_{h\to 0} \frac{2h(f(h)-4)}{(g(h)-4)^2} = \lim_{h\to 0} \frac{\frac{2(f(h)-4)}{h}}{\left[\frac{g(h)-4}{h}\right]^2}$$
$$= 2\lim_{h\to 0} \frac{\frac{f(h)-f(0)}{h}}{\left[\frac{g(h)-g(0)}{h}\right]^2}$$
$$= 2\cdot\frac{f'(0)}{[g'(0)]^2} = -\frac{2}{9}.$$

(10) 3. The graph of a function f is shown below. Use this graph to estimate $f'(-2)$, $f'(-1)$, $f'(0)$, $f'(1)$, and $f'(2)$. (If any of these derivatives don't exist, explain why.) Then sketch the graph of the derivative function f'.





$f'(-2) \approx -\frac{2}{3}$

$f'(-1) \approx -1$

$f'(0)$ does not exist because f is not continuous at 0

$f'(1) \approx \frac{4}{5}$

$f'(2) \approx 2$

(8) 4. Find the equation of the tangent line to the curve $y = 5x/(\sin x + \cos x)$ at the point $(\pi. -5\pi)$.

Let $f(x) = \dfrac{5x}{\sin x + \cos x}$.

The slope of the tangent line at $(\pi, -5\pi)$ is $f'(\pi)$.

$$f'(x) = \frac{5(\sin x + \cos x) - 5x(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$f'(\pi) = \frac{-5 - 5\pi(-1)}{1} = 5\pi - 5.$$

point-slope form of line: $(y + 5\pi) = (5\pi - 5)(x - \pi)$.

(12) 5. A particle is traveling along the x-axis. Its position at time t is given by $s(t) = (t^2 - 3)e^t$, $-\infty < t < \infty$.

a. Find all times when the instantaneous velocity of the particle is 0.

b. Find all times when the particle is moving to the left.

$$s'(t) = 2te^t + (t^2-3)e^t = (t^2+2t-3)e^t.$$

a. $s'(t) = 0$ when $t^2+2t-3 = 0$.

$(t^2+2t-3) = (t+3)(t-1)$, so $s'(t)=0$ when $t=1$ and $t=-3$

b. the particle is moving to the left when $s'(t) < 0$. This occurs when $(t+3)(t-1) = t^2+2t-3 < 0$, which happens for $-3 < t < 1$.