Thanks for the pointer to your draft! I think that it is proves the “2-step” of Dodgson nicely. Now the problem is to figure out how to make induction work...

Kiran and I haven’t finished our paper, mostly because I’ve been dallying hoping to prove more than we have. The result of that effort is that I have a clearer understanding of how the cases that we can prove are separated from the general case (and the gap is further than I thought...). Also I have to say that Kiran isn’t convinced that the totally general algebraic conjecture is true (he is much more favorably disposed towards the DVR version of the conjecture).

Anyway, I regard the sequence Nomos-6 (Kiran’s name) as a key test case; the recursion is

$$x_n x_{n+6} = x_{n+1} x_{n+5} + x_{n+2} x_{n+4}.$$

The empirical evidence for generalized Robbins (i.e., over a DVR) is enormous. The evidence for the algebraic conjecture is nontrivial but (for understandable reasons) not nearly as extensive. It *seems* that an “elementary” proof of Laurentness, such as the proof of Laurentness for Somos-4 (i.e., a proof that works over a DVR), does not exist for Nomos-6. (I’d love to be proved wrong about that...)

If that’s right then perhaps the most natural tack is to modify the Laurentness proof in Clusters III to include epsilons. Unfortunately, that proof uses the UFD property of the underlying ring in one key place, and the ring with epsilons is *not* a UFD.

Meanwhile, The paper ”Electroid varieties and a compactification of the space of electrical networks” by Thomas Lam (on the arXiv) has many interesting properties: (a) it heavily references some of your earlier work on the topic, (b) it uses the term ”juggling pattern” in a serious way (referring to a certain kind of affine permutation of the integers), and (c) it has a heavy cluster feel, both because Lam has done some nice work with cluster algebras and because some of the ideas relate to things we know and love. Anyway, I thought that you’d be amused by the juxtaposition.

ANOTHER E-MAIL: Well, here’s one thing that “should” be merely a generalized version of the Robbins calculation, with errors, that you did. For the sake of specificity, I will state it for the case of Nomos-6, but it should work for any initial transition function $F(x_1, ..., x_n)$ that is a sum of two monomials M_1 and M_2 that are monomials in disjoint subsets of $x_1, ..., x_n$. [I’ll explain this remark more fully below.]

Consider the 4-vertex graph

s w

$t \longrightarrow u$

Each vertex has a cluster (ordered 6-tuple) associated to it:

$$\begin{array}{rcl} s & : & (x_1, x_2, x_3, x_4, x_5, x_6) \\ t & : & (y_1, x_2, x_3, x_4, x_5, x_6) \\ u & : & (y_1, y_2, x_3, x_4, x_5, x_6) \\ w & : & (z_1, y_2, x_3, x_4, x_5, x_6) \end{array}$$

where $x_1, ..., x_6$ are indeterminates, and y_1, y_2, z_1 are defined by

$$\begin{array}{rcl} x_1 y_1 & = & x_2 x_6 + x_3 x_5 \\ x_2 y_2 & = & x_3 y_1 + x_4 x_6 \\ y_1 z_1 & = & y_2 x_3 x_5 + x_4 x_6^2. \end{array}$$

Given monomials F_1 and F_2 (associated to the transitions $s \to t$ and $t \to w$) then the transition $u \to w$ (a mutation in the direction 1) can be obtained from F_1 and F_2 by the following procedure: Think of the mutation polynomials as being polynomials in indeterminates X_1 through X_6. Replace X_1 by 0 in F_2 and divide by X_2; substitute this expression into F_1, and multiply by the unique monomial that produces a sum of coprime monomials.

So in the above example F_1 is $X_2X_6 + X_3X_5$, and F_2 is $X_1X_3 + X_4X_6$. The result of setting $X_1 = 0$ in F_2, dividing by X_2, and substituting, is

$$(X_4X_6/X_2)X_6 + X_3X_5.$$

Normalizing gives

$$X_4X_6^2 + X_2X_3X_5.$$

The Dodgson condensation calculation should have a similar transition at its core.

It is not possible to start with an arbitrary F_1, F_2 — they have to have a certain compatibility relationship. I'd be happy to chat about this on the phone if that would be useful.

My belief is that they (Somos and Robbins) are "the same." Note that the proof of Laurentness for condensation is somewhat elaborate, as is the proof of Laurentness for Somos sequences (when it's true). The "simple" proof of Laurentness for Somos-4 follows holds whenever the recursion has a specific property, which neither condensation nor Nomos-6 have.