A Table of Isoperimetric Ratios

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For a simple closed plane curve, the **isoperimetric ratio** is

$$\frac{P^2}{A}$$

where P is the length of the curve, and A is the area enclosed by the curve. This ratio is a dimensionless constant that is invariant under scaling: changing the *size* of the shape does not change this ratio.

The table below gives this ratio for a variety of shapes.

One can prove that the circle is the plane curve with the smallest (and hence, smallest possible) isoperimetric ratio, and so the table begins with the circle.

I started this table as a result of a homework problem in our precalculus class at the University of Washington. The problem asks the student to consider a situation in which we have a piece of wire of known length; we want to cut the wire and bend one resulting piece into a circle and the other piece into a square. The question is: where should we cut the wire so that the area of the circle plus the area of the square is minimized?

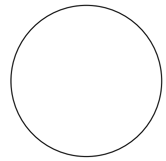
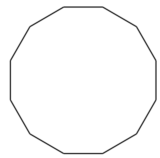
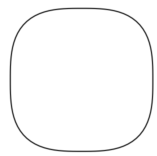
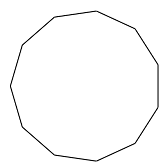
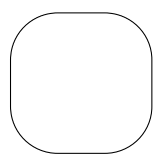
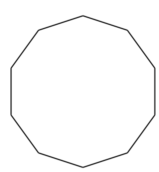
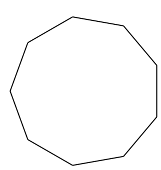
When writing exam questions of a similar flavor, the relative "efficiencies" of different planar shapes come under consideration.

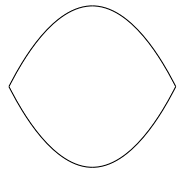
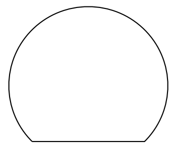
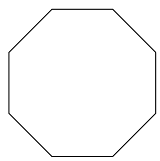
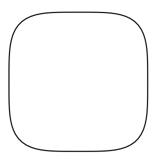
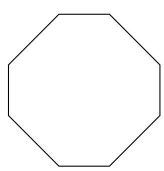
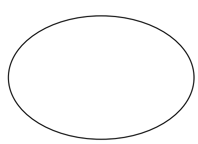
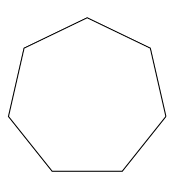
For any wire length, if the two shapes have isoperimetric ratios r_1 and r_2, then the wire should be cut into two pieces with lengths in the ratio $r_1 : r_2$ if we wish to minimize the total area.

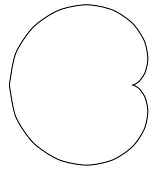
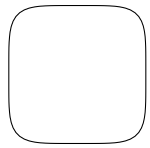
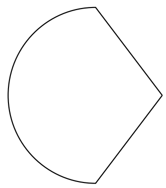
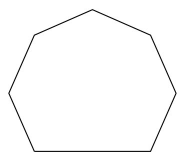
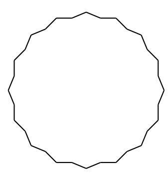
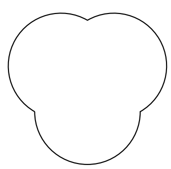
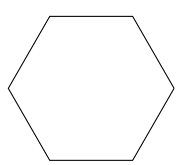
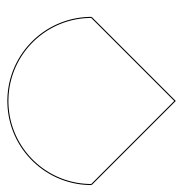
Cited by

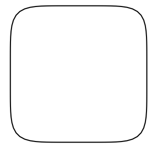
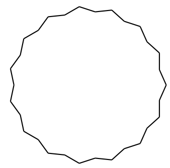
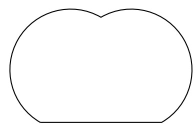
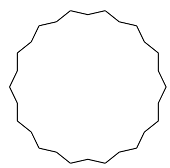
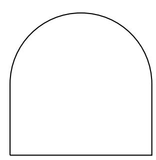
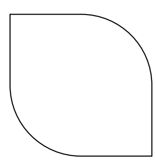
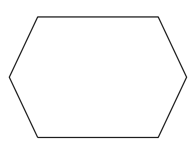
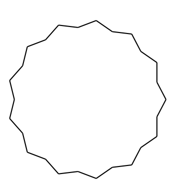
This table is cited in the following paper:

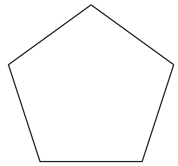
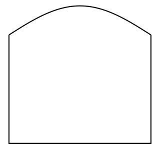
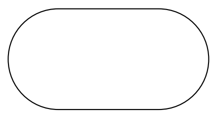
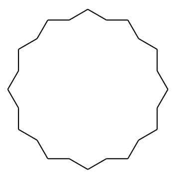
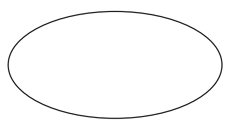
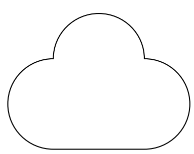
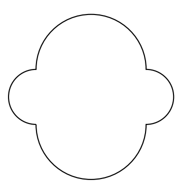
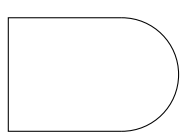
Hirvonen, Petri & Boissonière, Gabriel & Fan, Zheyong & Achim, Cristian & Provatas, Nikolas & Elder, Ken & Ala-Nissila, Tapio. (2018). Grain extraction and microstructural analysis method for two-dimensional poly and quasicrystalline solids. Physical Review Materials. 2. 10.1103/PhysRevMaterials.2.103603.

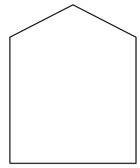
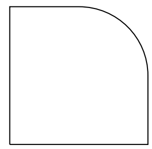
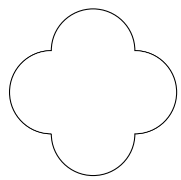
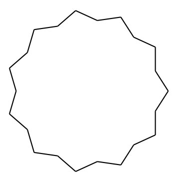
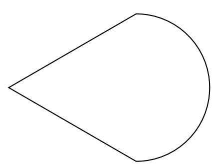
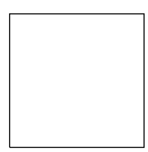
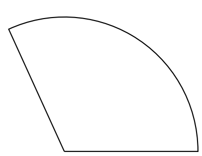
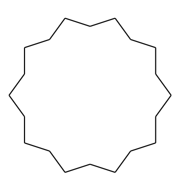
circle		$4\pi \approx 12.56637061$
regular 12-gon		$96 - 48\sqrt{3} \approx 12.86156124$
super ellipse $\lvert x\rvert^3 + \lvert y\rvert^3 = 1$		≈ 12.87612775
regular 11-gon		$44 \tan \dfrac{\pi}{11} \approx 12.91956568$
square with quarter circle rounded corners (radius of corners is one-third of overall width)		$\dfrac{(4+2\pi)^2}{5+\pi} \approx 12.98810988$
regular 10-gon		$40 \tan \dfrac{\pi}{10} \approx 12.99678784$
regular 9-gon		$36 \tan \frac{\pi}{9} \approx 13.10292843$

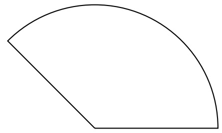
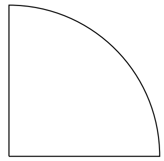
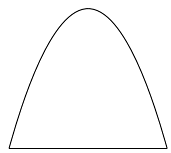
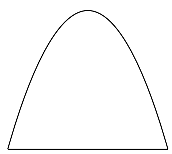
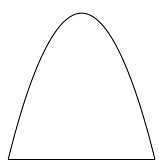
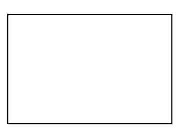
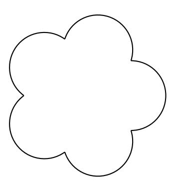
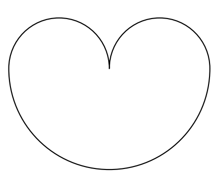
optimum region bounded by symmetric parabolic arcs (parabolas are approximately $y = 0.97300151x^2$ on the interval $(-0.97300151, 0.97300151)$		≈ 13.11934320
three-quarter truncated circular arc		≈ 13.14170265
regular octagon (equivalent to square augmented by four optimal isosceles triangles)		$32(\sqrt{2}-1) \approx 13.25483399$
super ellipse $\|x\|^4 + \|y\|^4 = 1$		≈ 13.28104122
square with hacked off corners (horizontal and vertical edges are one-third overall width)		$\dfrac{(4+4\sqrt{2})^2}{7} \approx 13.32211914$
ellipse, 3:2		≈ 13.35374554
regular septagon		$28 \tan \dfrac{\pi}{7} \approx 13.48408932$

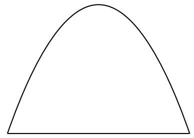
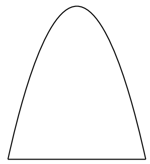
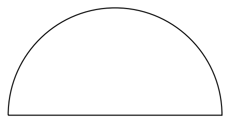
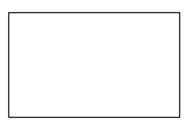
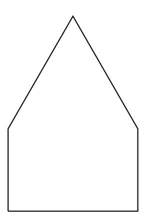
cardioid		$\frac{128}{3\pi} \approx 13.58122181$
super ellipse $\lvert x\rvert^5 + \lvert y\rvert^5 = 1$		≈ 13.61890282
semicircle with optimal adjoined isosceles triangle		≈ 13.71361987
square with three optimal adjoined isosceles triangles		≈ 13.75752837
16 pointed star		≈ 13.77927647
equilateral triangle with three adjoined semicircles		≈ 13.78342300
regular hexagon		$\frac{24}{\sqrt{3}} \approx 13.85640646$
semicircle with adjoined right isosceles triangle		$\frac{2(2\sqrt{2}+\pi)^2}{\pi+2} \approx 13.86385058$

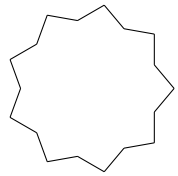
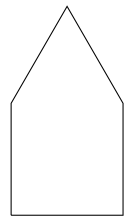
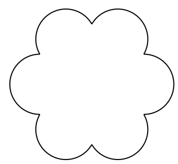
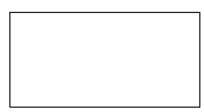
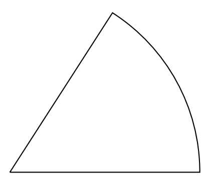
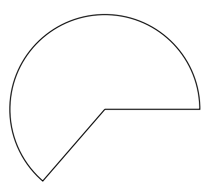
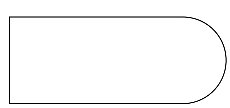
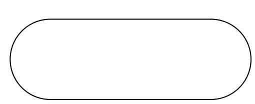
super ellipse $\|x\|^6 + \|y\|^6 = 1$		≈ 13.88994096
15 pointed star		≈ 13.96032741
equilateral triangle with two adjoined semicircles		$\dfrac{4(1+\pi)^2}{\pi+\sqrt{3}} \approx 14.07800126$
14 pointed star		≈ 14.18654408
semicircle with optimal adjoined rectangle		$8 + 2\pi \approx 14.2831853071$
square with two equally, optimally rounded corners (radii are half the side length)		$8 + 2\pi \approx 14.2831853071$
square with two optimal adjoined isosceles triangles		≈ 14.35072219
13 pointed star		≈ 14.47485646

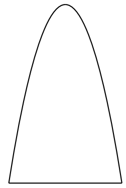
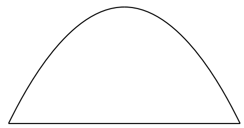
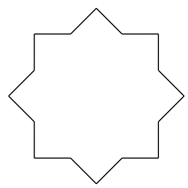
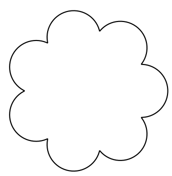
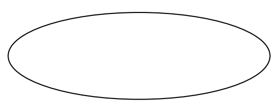
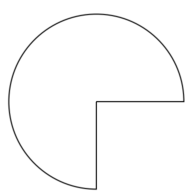
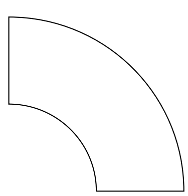
regular pentagon		$\frac{100}{\sqrt{25+10\sqrt{5}}} \approx 14.53085056$
optimal region bounded by $\frac{1}{x^2+1}$, the x-axis, and $x = \pm a$, where $a \approx 0.5181167$		≈ 14.695335322
square with two adjoined semicircles		$\frac{4(2+\pi)^2}{4+\pi} \approx 14.80676722$
12 pointed star		≈ 14.85125168
ellipse, 2:1		≈ 14.93924249
square with three adjoined semicircles		$\frac{2(2+3\pi)^2}{8+3\pi} \approx 14.98160283$
rectangle, 2:1, with four adjoined semicircles		$\frac{36\pi^2}{8+5\pi} \approx 14.98676855$
square with adjoined semicircle		$\frac{2(6+\pi)^2}{8+\pi} \approx 15.00121550$

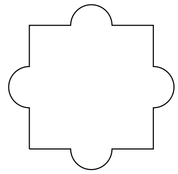
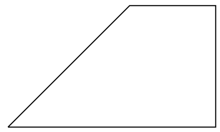
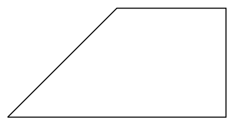
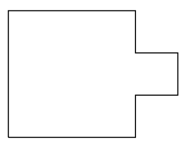
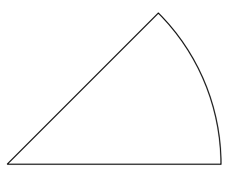
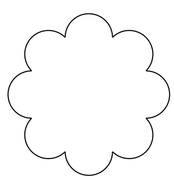
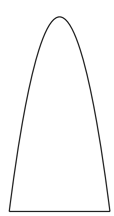
square with optimal adjoined isosceles triangle		≈ 15.07344594
square with single optimally rounded corner (radius is one half the side length)		$12 + \pi \approx 15.14159265$
square with four adjoined semicircles		$\frac{8\pi^2}{2+\pi} \approx 15.35649370$
11 pointed star		≈ 15.35751730
equilateral triangle with adjoined semicircle		$\frac{2(4+\pi)^2}{\pi + 2\sqrt{3}} \approx 15.44193344$
square		16
circular sector, two radians (optimal circular sector)		16
10 pointed star		≈ 16.06491327

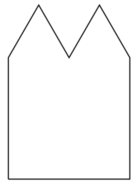
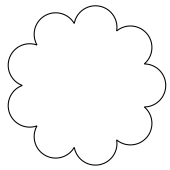
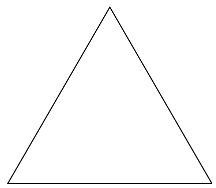
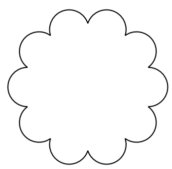
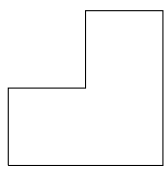
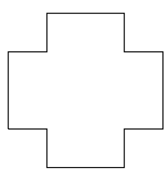
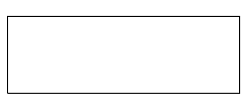
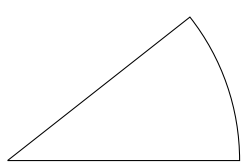
circular sector, $3\pi/4$ radians		$\dfrac{(8+3\pi)^2}{6\pi} \approx 16.10769443$
quarter circular sector		$\dfrac{4(2+\frac{\pi}{2})^2}{\pi} \approx 16.23455083$
optimal parabolic chunk (2 by ~ 1.7685879; region bounded by $y=1$ and $y \approx 3.127903x^2$)		≈ 16.48263518
parabolic chunk: region bounded by $y=x^2$ and $y=3$		≈ 16.48509413
parabolic chunk: region bounded by $y=x^2$ and $y=4$		≈ 16.56740039
rectangle, 3:2 sides		$\dfrac{50}{3} = 16.\bar{6}$
regular pentagon with adjoined semicircles		$\dfrac{50\pi^2}{5\pi + 2\sqrt{25+10\sqrt{5}}} \approx 16.74415928$
semicircle augmented by two semicircles, I		$\dfrac{16}{3}\pi \approx 16.75516081$

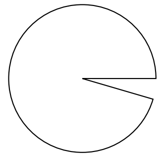
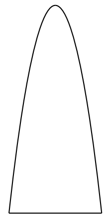
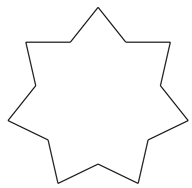
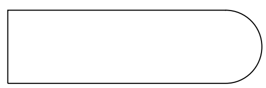
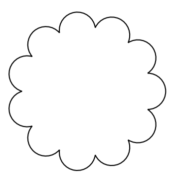
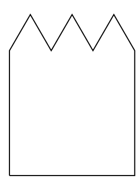
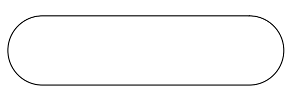
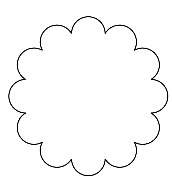
parabolic chunk: region bounded by $y = x^2$ and $y = 2$		≈ 16.76938845
parabolic chunk: region bounded by $y = x^2$ and $y = 5$		≈ 16.79033103
semicircle		$\dfrac{2(2+\pi)^2}{\pi} \approx 16.82966439$
golden rectangle (rectangle with side ratio 1:ϕ where $\phi = \frac{1}{2}(1+\sqrt{5})$)		$8\phi + 4 \approx 16.944271909$
rectangle with side ratio 1:1.63712909 . . .		≈ 16.99181787
rectangle with side ratio 1:1.63712909 . . . with adjoined semicircles		≈ 16.99181787
equilateral triangle with adjoined optimal rectangle		$4(6-\sqrt{3}) \approx 17.07179676$

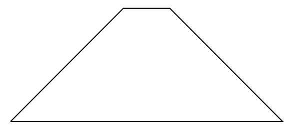
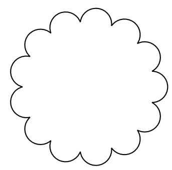
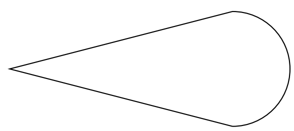
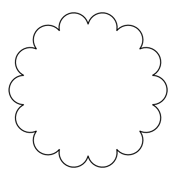
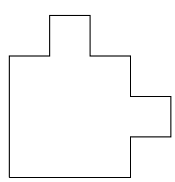
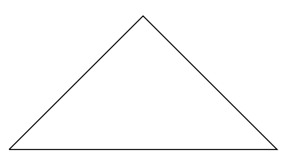
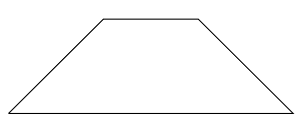
9 pointed star		≈ 17.10465828
square with adjoined equilateral triangle		$\frac{100}{4+\sqrt{3}} \approx 17.44576302$
regular hexagon with adjoined semicircles		$\frac{12\pi^2}{\pi + 2\sqrt{3}} \approx 17.92926648$
rectangle, 2:1		18
circular sector, one radian		18
circular sector, four radians		18
rectangle, 2:1, with one adjoined semicircle		$\frac{2(10+\pi)^2}{16+\pi} \approx 18.044627800654$
rectangle, 2:1, with two adjoined semicircles		$\frac{4(4+\pi)^2}{8+\pi} \approx 18.31061221$

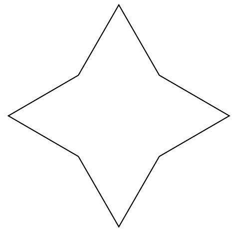
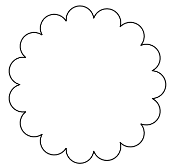
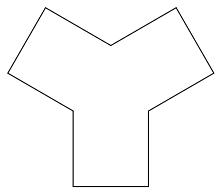
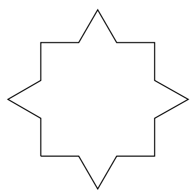
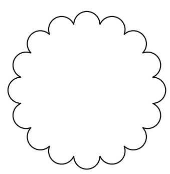
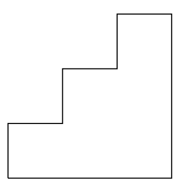
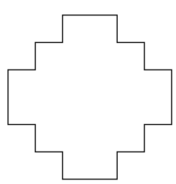
parabolic chunk: region bounded by $y = x^2$ and $y = 10$		≈ 18.38876821
parabolic chunk: region bounded by $y = x^2$ and $y = 1$		≈ 18.43547307
8 pointed star		$64 - 4^{11/4} \approx 18.74516600$
regular septagon with adjoined semicircles		≈ 18.94192830
ellipse, 3:1		≈ 18.95220996
three-quarter circular sector		$\frac{(4+3\pi)^2}{3\pi} \approx 19.12243068$
quarter circular sector with half-radius quarter circular sector removed		$\frac{(4+3\pi)^2}{3\pi} \approx 19.12243068$

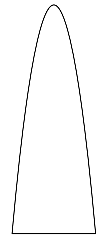
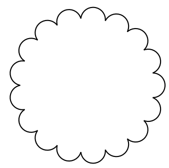
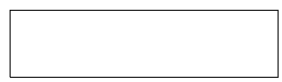
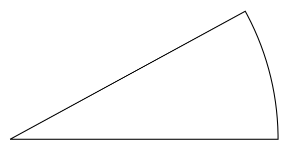
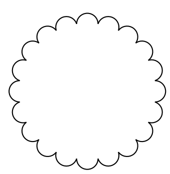
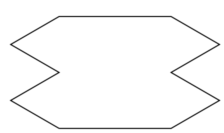
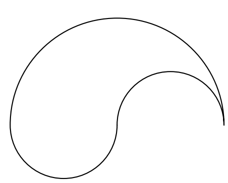
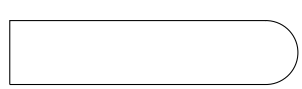
square with an adjoined semicircle on each side's middle third		$\frac{8(\pi+4)^2}{18+\pi} \approx 19.29933907$
isosceles right triangle with optimal adjoined rectangle $(1 : \frac{\sqrt{2}}{2})$		$8+8\sqrt{2} \approx 19.31370849$
isosceles right triangle with adjoined square		$\frac{2(4+\sqrt{2})^2}{3} \approx 19.54247233$
square with square adjoined to middle-third of one side		$\frac{98}{5} = 19.6$
circular sector, $\pi/4$ radians		$\frac{(8+\pi)^2}{2\pi} \approx 19.756712684$
regular octagon with adjoined semicircles		≈ 19.81346029
parabolic chunk: region bounded by $y = x^2$ and $y = 15$		≈ 20.00749094

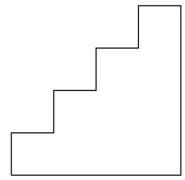
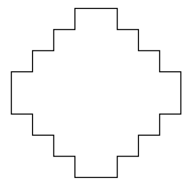
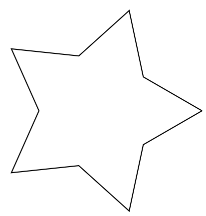
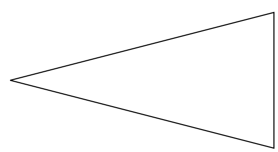
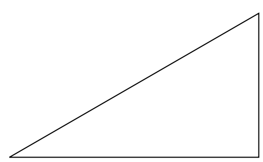
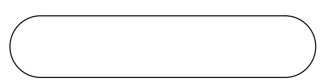
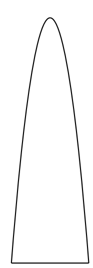
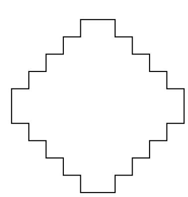
square with two equilateral triangles adjoined to one side		$\frac{200}{8+\sqrt{3}} \approx 20.55065309$
regular nonagon with adjoined semicircles		≈ 20.569895
equilateral triangle		$\frac{36}{\sqrt{3}} \approx 20.78460969$
regular decagon with adjoined semicircles		≈ 21.231897
two step unit staircase		$\frac{64}{3} = 21.\bar{3}$
square with quarter-sized squares removed from each corner (aka order-2 Aztec diamond)		$\frac{64}{3} = 21.\bar{3}$
rectangle, 3:1		$\frac{64}{3} = 21.\bar{3}$
circular sector, $2/3$ radians		$\frac{64}{3} = 21.\bar{3}$

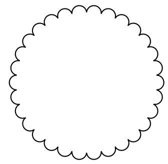
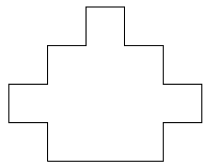
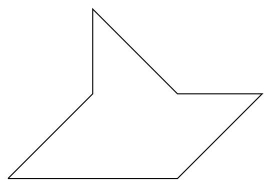
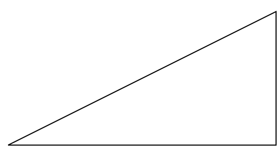
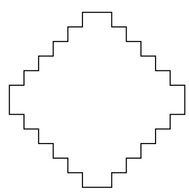
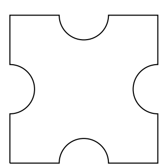
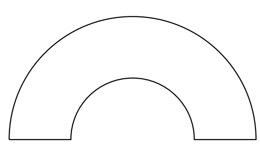
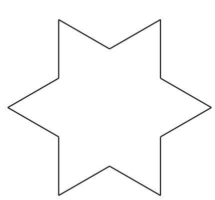
circular sector, 6 radians		$\frac{64}{3} = 21.\bar{3}$
parabolic chunk: region bounded by $y = x^2$ and $y = 20$		≈ 21.50079173
7 pointed star		≈ 21.62680012
rectangle, 3:1, with one adjoined semicircle		$\frac{2(14+\pi)^2}{24+\pi} \approx 21.65194964$
regular 11-gon with adjoined semicircles		≈ 21.81573534
square with three equilateral triangles adjoined to one side		$\frac{300}{12+\sqrt{3}} \approx 21.84670041$
rectangle, 3:1, with two adjoined semicircles		$\frac{4(6+\pi)^2}{12+\pi} \approx 22.076598718$
regular 12-gon with adjoined semicircles		≈ 22.33427625

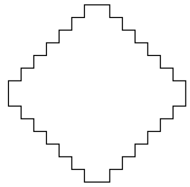
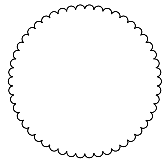
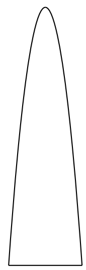
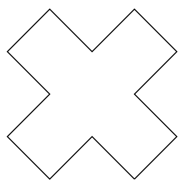
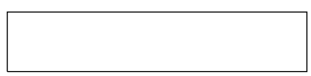
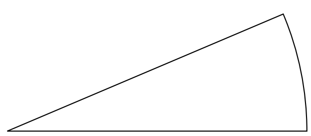
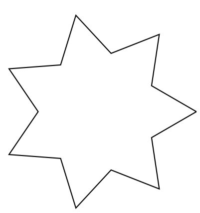
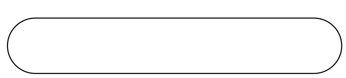
optimal "45 degree" trapezoid (base= 1, height= $\frac{1}{\sqrt{2}+1}$) (aka optimal truncated isosceles right triangle aka optimal rectangle with two adjoined isosceles right traingles)		$16\sqrt{2} \approx 22.62741699796$
regular 13-gon with adjoined semicircles		≈ 22.79777267
isosceles triangle, 2:1 sides, with adjoined semicircle		$\frac{2(8+\pi)^2}{\pi + 2\sqrt{15}} \approx 22.80310635$
regular 14-gon with adjoined semicircles		≈ 23.21447316
square with squares adjoined to middle third of two sides		$\frac{256}{11} = 23.\overline{27}$
isosceles right triangle		$2(2+\sqrt{2})^2 \approx 23.31370849$
square with two adjoined isosceles right triangles		$2(2+\sqrt{2})^2 \approx 23.31370849$

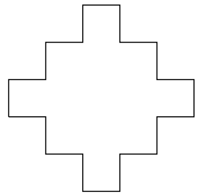
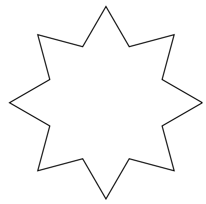
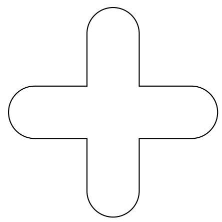
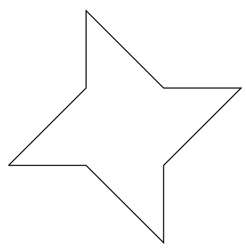
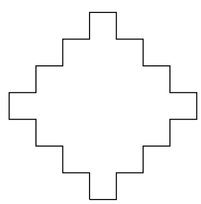
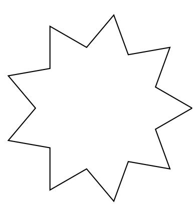
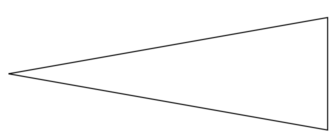
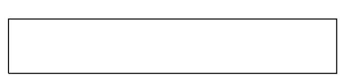
square with four adjoined equilateral triangles		$\frac{64}{1+\sqrt{3}} \approx 23.42562584$
regular 15-gon with adjoined semicircles		≈ 23.59107664
equilateral triangle with three adjoined squares		$\frac{324}{12+\sqrt{3}} \approx 23.59443644$
square with an adjoined equilateral triangles on each side's middle third		$\frac{256}{9+\sqrt{3}} \approx 23.85378196$
regular 16-gon with adjoined semicircles		≈ 23.93307364
three-step unit staircase		24
order-3 Aztec diamond		24

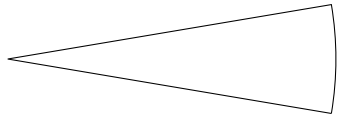
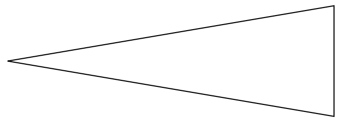
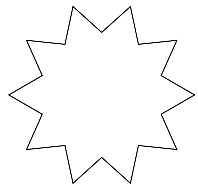
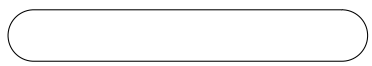
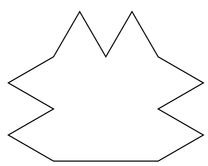
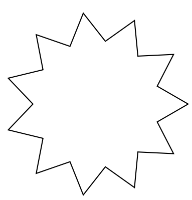
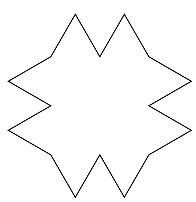
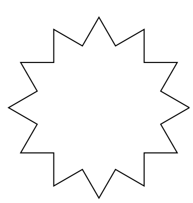
parabolic chunk: region bounded by $y = x^2$ and $y = 30$		≈ 24.15608124
regular 17-gon with adjoined semicircles		≈ 24.2450034
rectangle, 4:1		25
circular sector, $1/2$ radian		25
regular 20-gon with adjoined semicircles		≈ 25.03530855
square with two equilateral triangles adjoined to two sides		$\dfrac{144}{4+\sqrt{3}} \approx 25.1218987469293$
semicircle augmented with two semicircles, II		$8\pi \approx 25.13274122$
rectangle, 4:1, with one adjoined semicircle		$\dfrac{2(18+\pi)^2}{32+\pi} \approx 25.43805821$

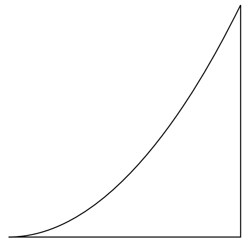
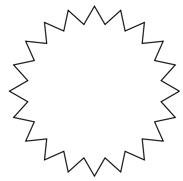
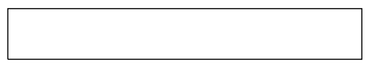
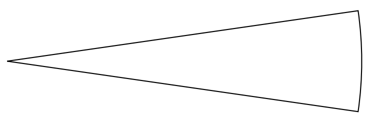
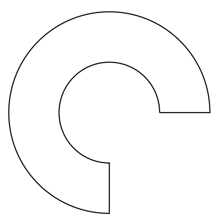
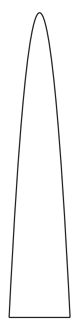
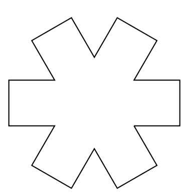
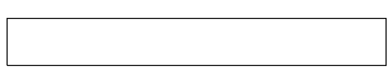
four-step unit staircase		$\frac{128}{5} = 25.6$
order-4 Aztec diamond		$\frac{128}{5} = 25.6$
regular pentagon with adjoined equilateral triangles		≈ 25.73644244
isosceles triangle, 2:1 sides		$\frac{100}{\sqrt{15}} \approx 25.81988897$
right triangle, 30-60-90		$12 + 8\sqrt{3} \approx 25.85640646$
rectangle, 4:1, with two adjoined semicircles		$\frac{4(8+\pi)^2}{16+\pi} \approx 25.94038836893$
parabolic chunk: region bounded by $y = x^2$ and $y = 40$		≈ 26.48326571
order-5 Aztec diamond		$\frac{80}{3} = 26.\bar{6}$

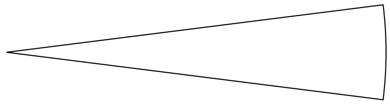
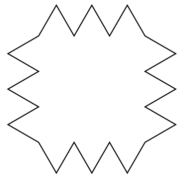
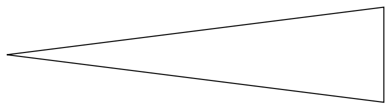
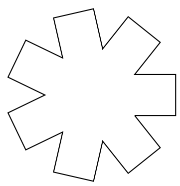
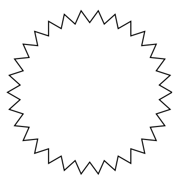
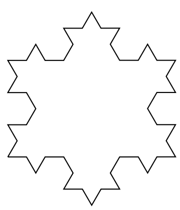
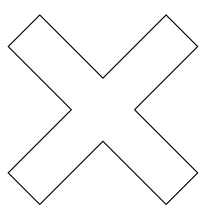
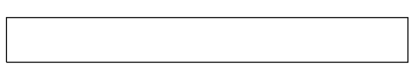
regular 30-gon with adjoined semicircles		≈ 26.71031462
square with squares adjoined to the middle thirds of three sides		27
square with three adjoined right isosceles triangles		$\frac{68+48\sqrt{2}}{5} \approx 27.17645019$
right triangle, 2:1 legs		$(3+\sqrt{5})^2 = 14+6\sqrt{5} \approx 27.41640786$
order-6 Aztec diamond		$\frac{192}{7} = 27.\overline{428571}$
square with one-third diameter semicircles removed from each side		$\frac{2(8+2\pi)^2}{18-\pi} \approx 27.46046434$
semicircle with half-diameter semicircle removed		$\frac{2(3\pi+2)^2}{3\pi} \approx 27.69838228$
6 pointed star (Koch curve, stage 2; regular hexagon with adjoined equilateral triangles)		$\sqrt{768} \approx 27.71281292$

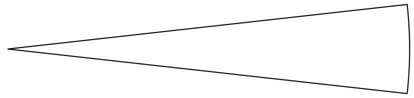
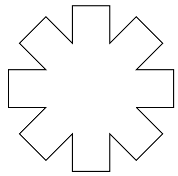
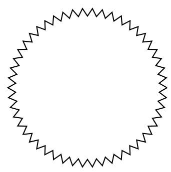
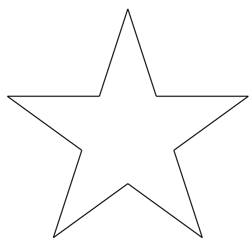
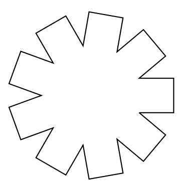
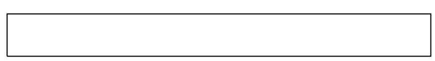
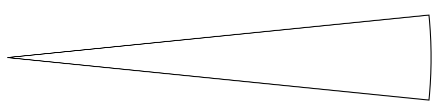
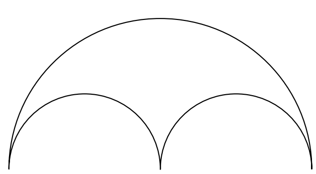
order-7 Aztec diamond		28
regular 50-gon with adjoined semicircles		≈ 28.25482853
parabolic chunk: region bounded by $y = x^2$ and $y = 50$		≈ 28.57579596
5-square cross		$\frac{144}{5} = 28.8$
rectangle, 5:1		$\frac{144}{5} = 28.8$
circular sector, $2/5$ radian		$\frac{144}{5} = 28.8$
regular septagon with adjoined equilateral triangles		≈ 29.40734584
rectangle, 5:1, with two adjoined semicircles		$\frac{4(10+\pi)^2}{20+\pi} \approx 29.851265651086$

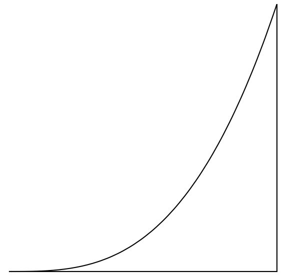
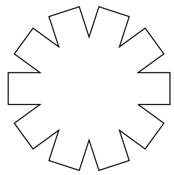
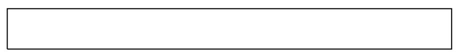
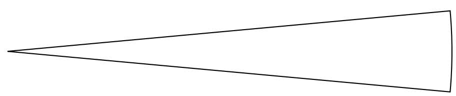
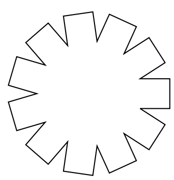
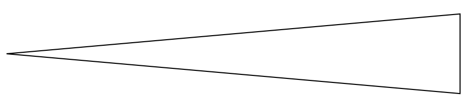
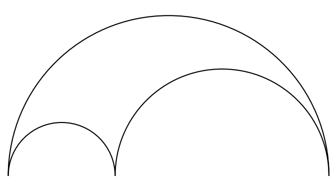
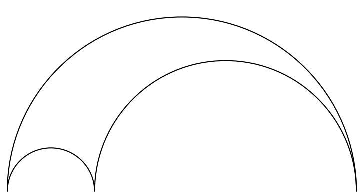
13-square diamond		$\frac{400}{13} \approx 30.76923076$
regular octagon with adjoined equilateral triangles		≈ 30.87116222
5-square cross with four adjoined semicircles		$\frac{8(4+\pi)^2}{\pi+10} \approx 31.04789318$
square with four adjoined isosceles right triangles		$16 + \frac{32}{3}\sqrt{2} \approx 31.08494466$
25-square diamond		$\frac{784}{25} = 31.36$
regular nonagon with adjoined equilateral triangles		≈ 32.14624235
isosceles triangle, 20° apex angle		≈ 32.21915602
rectangle, 6:1		$\frac{98}{3} = 32.\bar{6}$

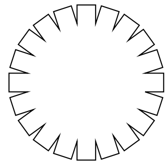
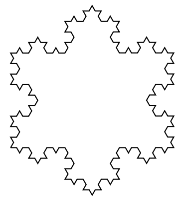
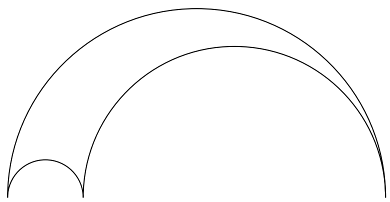
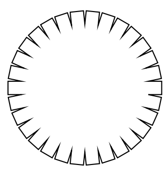
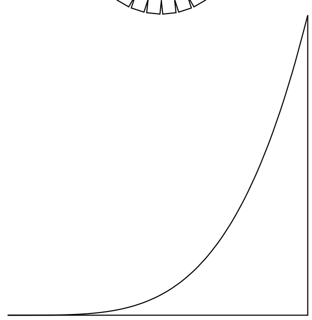
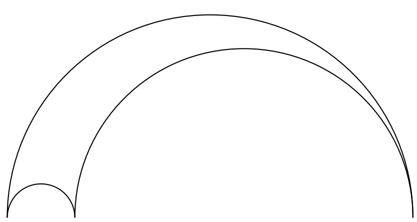
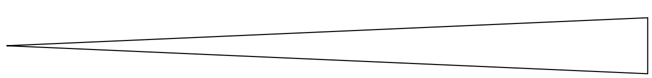
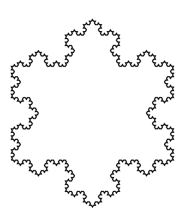
circular sector, 1/3 radian		$\frac{98}{3} = 32.\bar{6}$
isosceles triangle, 3:1 sides		$\frac{196}{\sqrt{35}} \approx 33.13004679$
regular decagon with adjoined equilateral triangles		≈ 33.26587053
rectangle, 6:1, with two adjoined semicircles		$\frac{(24+2\pi)^2}{24+\pi} \approx 33.78841190$
square with two adjoined equilateral triangles on three sides		$\frac{450}{8+3\sqrt{3}} \approx 34.10084891$
regular 11-gon with adjoined equilateral triangles		≈ 34.25632027
square with two adjoined equilateral triangles on each side		$\frac{128}{2+\sqrt{3}} \approx 34.29749663$
regular 12-gon with adjoined equilateral triangles		≈ 35.13843876

region bounded by $y = x^2$, $y = 0$, $x = 1$		≈ 36.30913022
regular 20-gon with adjoined equilateral triangles		≈ 35.13843876
rectangle, 7:1		$\frac{256}{7} \approx 36.57142857$
circular sector, $2/7$ radian		$\frac{256}{7} \approx 36.57142857$
three-quarter circular sector with half-radius three-quarter circular sector removed		$\frac{(9\pi + 4)^2}{9\pi} \approx 36.84021812$
parabolic chunk: region bounded by $y = x^2$ and $y = 100$		≈ 36.99450714
regular hexagon with adjoined squares		≈ 37.682848
rectangle, 8:1		$\frac{81}{2} = 40.5$

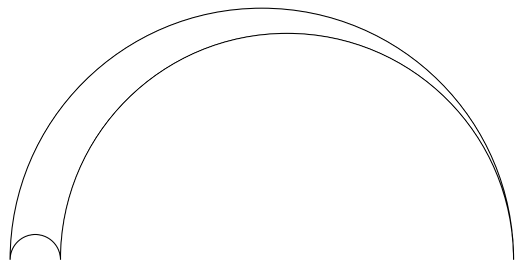
circular sector, 1/4 radian		$\frac{81}{2} = 40.5$
square with three equilateral triangles adjoined to each side		$\frac{192}{3+\sqrt{3}} = 40.57437416$
isosceles triangle, 4:1 sides		$\frac{324}{\sqrt{63}} \approx 40.82016308$
regular septagon with adjoined squares		≈ 41.471095
regular 30-gon with adjoined equilateral triangles		≈ 42.68026645
Koch curve, stage 3		$\frac{384}{5\sqrt{3}} \approx 44.34050067$
9-square cross		$\frac{400}{9} = 44.\bar{4}$
rectangle, 9:1		$\frac{400}{9} = 44.\bar{4}$

circular sector, $2/9$ radian		$\frac{400}{9} = 44.\bar{4}$
regular octagon with adjoined squares		≈ 44.900282
regular 50-gon with adjoined equilateral triangles		≈ 45.38596197
5 pointed star		$20\sqrt{10-2\sqrt{5}} \approx 47.02282018$
regular nonagon with adjoined squares		≈ 48.017945
rectangle, 10:1		$\frac{242}{5} = 48.4$
circular sector, $1/5$ radian		$\frac{242}{5} = 48.4$
semicircle augmented by two semicircles, III (arbelos)		$16\pi \approx 50.26548245$

region bounded by $y = x^3$, $y = 0$ and $x = 1$		≈ 50.34940281
regular decagon with adjoined squares		≈ 50.864099
rectangle, 11:1		$\frac{576}{11} = 52.\overline{36}$
circular sector, $2/11$ radian		$\frac{576}{11} = 52.\overline{36}$
regular 11-gon with adjoined squares		≈ 53.472417
isosceles triangle, 10° apex angle		≈ 54.45067722
arbelos, one-third and two-thirds arcs		$18\pi \approx 56.54866776$
arbelos, one-quarter and three-quarter arcs		$\frac{64\pi}{3} \approx 67.02064328$

regular 20-gon with adjoined squares		≈ 69.80970978
Koch curve, stage 4		$\frac{6144}{47\sqrt{3}} \approx 75.47319263$
arbelos, one-fifth and four-fifths arcs		$25\pi \approx 78.53981634$
regular 30-gon with adjoined squares		≈ 79.91496779
region bounded by $y = x^4$, $y = 0$ and $x = 1$		≈ 80.73353354
arbelos, one-sixth and five-sixths arcs		$\frac{144\pi}{5} \approx 90.47786842$
isosceles triangle, 5° apex angle		≈ 99.97197122
Koch curve, stage 5		$\frac{98304}{431\sqrt{3}} \approx 131.68408553$

arbelos, one-tenth and nine-tenths arcs



$$\frac{400\pi}{9} \approx 139.62634016$$

General formulas

Rectangles

For a rectangle with aspect ratio m (i.e., the ratio of non-equal length sides is $m : 1$), the ratio is

$$\frac{(2m+2)^2}{m} = 4m + 8 + \frac{4}{m}.$$

Right triangles

For a right triangle with non-right angle θ, the ratio is

$$\frac{2(1+\sin\theta+\cos\theta)^2}{\sin\theta\cos\theta}.$$

Isosceles triangles

For an isosceles triangle in which the sides with the same length are l times the length of the other side, the ratio is

$$\frac{4(2l+1)^2}{\sqrt{4l^2-1}}.$$

For an isosceles triangle with an "apex" angle of θ, the ratio is $4\tan\frac{\theta}{2}\left(1+\csc\frac{\theta}{2}\right)^2$.

Rectangle with adjoined semicircles

For a rectangle with aspect ratio m with semicircles adjoined to the "1" sides, the ratio is

$$\frac{(4m+2\pi)^2}{4m+\pi}.$$

For a value of $m \approx 1.637129085772..$, this is equal to the ratio for the $m : 1$ rectangle. If $m > 1.637129085772..$, adjoining the semicircles decreases the ratio, while for smaller m, adjoining semicircles increases the ratio.

Regular polygons

For a regular polygon with n sides, the ratio is $4n\tan\frac{\pi}{n}$.

Circular sectors

For a circular sector with angle θ, the ratio is

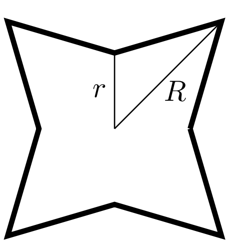
$$\frac{2(2+\theta)^2}{\theta}$$

This has a minimum of 16 at $\theta = 2$.

Polygonal Stars

For an n-sided polygonal star with equal side lengths, like the one shown, with inner radius r and outer radius R, the area is $\frac{1}{2}nrR\sin\frac{2\pi}{n}$ and the perimeter is $n\sqrt{r^2+R^2-2rR\cos\frac{2\pi}{n}}$ so the isoperimetric ratio is

$$\frac{2n(r^2+R^2-2rR\cos\frac{2\pi}{n})}{rR\sin\frac{2\pi}{n}}.$$



A special case of this are stars based on a regular polygon with $m \geq 5$ sides by extending the sides of the polygon until they intersect. In this case, we have $n = 2m$ and $R = r(\cos\frac{\pi}{m} + \sin\frac{\pi}{m}\tan\frac{2\pi}{m})$. In the table, such stars are called *n-pointed stars*.

Regular polygons with adjoined equilateral triangles

If we adjoin equilateral triangles to the sides of a regular n-gon, the resulting figure has an isoperimetric ratio of

$$\frac{16n}{\sqrt{3}+\cot\frac{\pi}{n}}.$$

As n tends to infinity, this approaches $16\pi = 50.265482\ldots$ from below.

Regular polygons with adjoined squares

If we adjoin squares to the sides of a regular n-gon, the resulting figure has an isoperimetric ratio of

$$\frac{36n}{4+\cot\frac{\pi}{n}}$$

As n tends to infinity, this approaches $36\pi = 113.097\ldots$ from below.

Regular polygons with adjoined semicircles

If we adjoin semicircles to the sides of a regular n-gon, the resulting figure has an isoperimetric ratio of

$$\frac{\pi^2 n}{\frac{\pi}{2}+\cot\frac{\pi}{n}}$$

As n tends to infinity, this approaches $\pi^3 \approx 31.0063\ldots$ from below.

n-square diamond/cross

The ratio is

$$\frac{(8m+12)^2}{2m^2+6m+5}$$

where m is the order of the diamond ($m=0$ is the 5-square cross, $m=1$ is the 13-square diamond, etc.)

This approaches 32 from below as m tends to infinity.

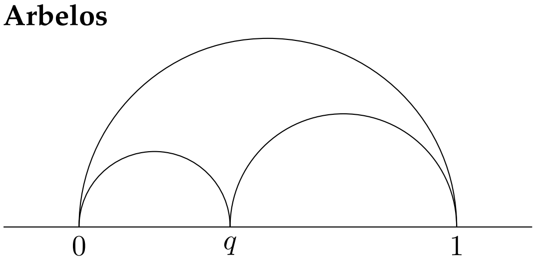
Koch curve

The ratio is

$$\frac{180\left(\frac{4}{3}\right)^{2n}}{\sqrt{3}(8-3\left(\frac{4}{9}\right)^{n})}$$

for the n-th iteration (i.e., $n=0$ is an equilateral triangle, $n=1$ is a six-pointed star, etc.)

Arbelos



The area is $\frac{\pi}{4}q(1-q)$ and the perimeter is π, so the ratio is

$$\frac{4\,\pi}{q(1-q)}.$$

Parabolic chunk

For regions bounded by a parabola and a line perpendicular to the parabola's axis, the ratio is

$$\frac{\left(2+\sqrt{1+4a^2}+\frac{1}{2a}\log(2a+\sqrt{1+4a^2})\right)^2}{\frac{4}{3}a}$$

where the parabola is $y = ax^2$ and the bounding line is $y = a$ (we can express all parabolic chunks in this form; note, for instance, that the shape bounded by $y = x^2$ and $y = b$ is the same as the shape bounded by $y = \sqrt{b}x^2$ and $y = \sqrt{b}$).

Aztec diamond

For an n-th order Aztec diamond, which looks like four copies of a set of n steps (of unit width and height) stuck together, the area is $2n(n+1)$ and the perimeter is $8n$, so the ratio is

$$\frac{P^2}{A} = \frac{64n^2}{2n(n+1)} = \frac{32n}{n+1}.$$

Incidentally, this is the same ratio as for a set of n unit steps themselves, as the perimeter is $4n$ and the area is $\frac{1}{2}n(n+1)$ which yields

$$\frac{P^2}{A} = \frac{(4n)^2}{\frac{1}{2}n(n+1)} = \frac{32n}{n+1}.$$

This is because the "stair" part and the non-stair part have the same length, so putting four of them together exactly doubles the perimeter while quadrupling the area (unlike what happens when you put, say, four quarter-circles together to make a circle).