Homework 4 - Math 381 A - Autumn 2015 - Dr. Matthew Conroy

1. Your friend goes out to a restaurant every Friday night. Your friend only likes to go to three different restaurants, let's call them 1, 2, and 3. The probability that they go to each restaurant on a Friday depends entirely on where they went the preceding Friday.

   After studying your friend's behavior, you come up with the following transition matrix:

$$A = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 0.3 & 0.1 & 0.6 \\ 0.7 & 0.2 & 0.1 \end{pmatrix}$$

   This says, for example, that if your friend goes to restaurant 3 this Friday, the probability that they go to restaurant 2 next Friday is 0.2.

   In the long run, what is the probability that your friend will go to each restaurant?

   That is, work out the stationary distribution of this Markov chain. Please give exact values, not decimal approximations.

2. You are playing a game with a fair, six-sided die.

   You start with a score of zero.

   On each turn, you roll the die once.

   If the die comes up 1, your score does not change.

   If the die comes up $2, 3, 4$ or 5, you add this value to your score.

   If the die comes up 6, you double your score.

   (a) What is the probability that you will reach a score of 20 or greater in 5 or fewer turns? Give an exact value.

   (b) How many turns, on average, will it take to reach a score of 20 or greater?
   In other words, what is the expected number of turns needed to reach a score of 20 or greater?
   You should use a theorem from lecture and give an exact value.
   (Example game. Your first roll is a 2, so your score is 2. Your second roll is 6, so your score is 4. Your third roll is 1, so your score is 4. Your fourth roll is 5, so your score is 9. Your fifth roll is 4, so your score is 13. Your sixth roll is 6, so your score exceeds 20. For this game, it took six rolls to reach a score of 20 or greater.)