Homework 2 - Math 381 A - Autumn 2015 - Dr. Matthew Conroy

1. Let a and b be integers. We say that a divides b iff $b = ak$ for some integer k.

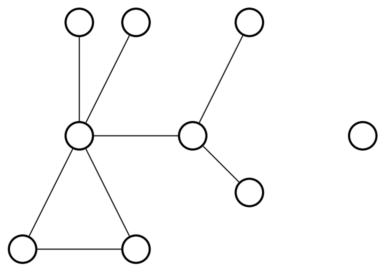
   So 2 divides 10, and 3 does not divide 8.

   Let $V = \{2, 3, 4, \ldots, 20\} = \{j \in \mathbb{Z} : 2 \leq j \leq 20\}$.

   Define a graph H with V as its vertex set and edge set E defined by $(v_1, v_2) \in E$ iff $v_1 \neq v_2$ and v_1 divides v_2 or v_2 divides v_1. So $(2, 6)$ is an edge in H; $(3, 4)$ is not.

   Recall that the edge $(2, 6)$ is the same as the edge $(6, 2)$.

   (a) The *degree sequence* of a graph G is the sequence of the degrees of all vertices in G. For example, the graph below has degree sequence $5, 3, 2, 2, 1, 1, 1, 1, 0$ (in decreasing order).

   Give the degree set of H in decreasing order.

   (b) A *path* from vertex u to vertex v in a graph G is an alternating sequence of vertices and edges

   $$u = u_0, e_1, u_1, e_2, \ldots, u_{n-1}, e_n, u_n = v$$

   where $e_i = (u_{i-1}, u_i)$ and none of the vertices are repeated.
   (A *walk* is the same as a path except that we allow repetition of edges and vertices.)
   The *length* of a path is the number of edges in the path.
   The *distance* from vertex u to vertex v in a graph G is the length of the shortest path from u to v.
   A graph G is connected if, for all pairs of vertices u and v in G, there is a path from u to v.
   Remove the vertices that have degree zero from H, to get the subgraph H'. Is H' connected?
   What two vertices in H' are farthest apart?

2. Let G be a graph with adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

   Use the fact that the ij-th entry of A^k gives the number of walks of length k from vertex i to vertex j in G to argue whether or not G is connected.