

Homework 2 - Math 300 D Winter 2014 - Dr. Matthew Conroy

1. Verify each of the following set identities by showing that the statement " x is in the left-hand set" is equivalent to the statement " x is in the right-hand set".
 - (a) $(A \cap B) \setminus A = \emptyset$
 - (b) $A \setminus (A \cap B) = A \setminus B$
 - (c) $(A \cap B) \setminus C = (A \setminus C) \cap B$
 - (d) $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$
 - (e) $C \setminus (A \cup B) = (C \setminus A) \setminus B$
2. Show that $P \Leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
3. Show that $(P \rightarrow Q) \wedge P$ is equivalent to $(P \wedge Q)$.
4. Show that $(P \rightarrow Q) \wedge (P \rightarrow R)$ is equivalent to $P \rightarrow (Q \wedge R)$.
5. Show that $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology.
6. Find a formula involving only \neg and \wedge that is equivalent to $P \Leftrightarrow Q$, and then find one involving only \neg and \rightarrow that is equivalent to $P \Leftrightarrow Q$.
7. Write useful contrapositives of the following sentences. Express the contrapositives as sentences, not as symbolic expressions.
 - (a) If x and y are real numbers, then $x + y$ is a real number.
 - (b) If x and y are integers, and at least one of them is even, then xy is even.
 - (c) If you earned at least 90% in my class, then you got an A.
 - (d) If it rains or snows, then I will go for a walk but I will not ride my bike.