Homework 8 - Math 300D - Autumn 2014 - Dr. Matthew Conroy

Relevant reading: Velleman 6.1, 6.2, 7.1, 7.2

1. Find the smallest $k \in \mathbb{Z}$ such that $n! > n^4$ for all $n \geq k$. Prove the result using induction.

2. Use induction to prove that
$$15 \mid 3^{4n} + 2^{12n+1} - 8$$
for all n in $\mathbb{Z}_{\geq 1}$.

3. Let n be a positive odd integer.

   Use induction to prove that the sum of all positive odd integers less than or equal to n is $\left(\frac{n+1}{2}\right)^2$.

4. If A and B are finite sets, and $A \cap B = \varnothing$, then $A \cup B$ is finite. In fact, if $|A| = n$ and $|B| = m$ and $A \cap B = \varnothing$, then $|A \cup B| = m + n$.

5. Let A be a finite set. Prove that if $f : A \to A$ is injective, then f is bijective.

6. Suppose A is an infinite set and B is a finite subset of A. Prove that $A \setminus B$ is infinite.

7. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.