Math 300 D - Autumn 2014 - Homework 6
Problems on functions
Relevant reading: Velleman, 4.6, 5.1, 5.2, 5.3

1. Suppose $f : A \to C$ and $g : B \to C$. Prove that if $A \cap B = \varnothing$, then $f \cup g : (A \cup B) \to C$.

2. Suppose R is a relation on a set A. Is it possible that R is both a function and an equivalence relation? Answer this question by completing and proving the statement "R is a function and an equivalence relation iff ...".

3. Let S and T be sets and $f : S \to T$. Define a relation R on S by
$$(a, b) \in R \Leftrightarrow f(a) = f(b).$$
Prove that R is an equivalence relation.

4. Define a function $f : \mathbb{R} \to \mathbb{R}$ by
$$f(x) = \begin{cases} 2x & \text{if } x \in \mathbb{Q} \\ -3x & \text{if } x \notin \mathbb{Q} \end{cases}$$
Is f one-to-one? Is f onto? Is f^{-1} a function? State and prove a theorem.

5. Let A, B and C be sets. Let $f : A \to B$ and $g : B \to C$.
   (a) Prove that if f and g are onto, then $g \circ f$ is onto.
   (b) Prove that if $g \circ f$ is onto, then g is onto.
   (c) If $g \circ f$ is onto, is f necessarily onto? Prove your answer.

6. Let A be the set of subsets of $\mathbb{R}$. Define a function $f : \mathbb{R} \to A$ by
$$f(x) = \{z \in \mathbb{R} : |z| > x\}.$$
Is f one-to-one? Is f onto?

7. Let A and B be sets, and $f : A \to B$. Suppose f is one-to-one. Prove that there exists a subset $C \subseteq B$ such that $f^{-1} : C \to A$.