Math 300 D - Autumn 2014 - Homework 5
Problems on relations and equivalence relations
Relevant reading: Velleman, 4.1, 4.2, 4.6 (Velleman says a ton more about relations in 4.3-4.5, if you are interested, but you won't need to know any of that stuff for this course)

1. Let $A = \{a, b, c, d\}$. Let $R = \{(a, a), (b, b), (c, c), (a, c), (c, a), (d, d)\}$. R is an equivalence relation. Give the set of equivalent classes of R, A/R. Note that A/R is a partition of A.

2. Let $A = \{a, b, c, d\}$. Let $P = \{\{a\}, \{b, c\}, \{d\}\}$. P is a partition of A. Give the equivalence relation R such that $A/R = P$.

3. How many equivalence relations are there on a set with three elements? List all of the equivalence relations.

4. For each of the following relations, determine whether it is reflexive, symmetric and transitive. Conclude whether or not the relation is an equivalence relation.

   (a) Let $A = \mathbb{R}$. Define a relation R on A by
   $$(x, y) \in R \Leftrightarrow x < y$$
   (b) Let $A = \mathbb{R}$. Define a relation R on A by
   $$(x, y) \in R \Leftrightarrow x \leq y$$
   (c) Let $A = \mathbb{R} \times \mathbb{R}$.
   Define a relation R on A by
   $((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow$ the distance from (x_1, y_1) to (x_2, y_2) is a rational number.
   (d) Let $A = \mathbb{R} \times \mathbb{R}$.
   Define a relation R on A by
   $((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow$ the distance from (x_1, y_1) to (x_2, y_2) is an irrational number.

5. Let $a, b \in \mathbb{Z}$. Let $m \in \mathbb{Z}_{>0}$.
   We say a is **congruent** to b mod m iff $m|(a - b)$.
   If a is congruent to b mod m, we write
   $$a \equiv b \pmod{m}.$$
   Prove that if $a \equiv b \pmod{m}$ and $c \equiv d(\operatorname{mod} m)$, then
   $$(a + c) \equiv (b + d) \pmod{m}$$
   and
   $$ac \equiv bd \pmod{m}$$

6. Let $A = \mathbb{R}$.
   Define a relation R on A by
   $$(a, b) \in R \Leftrightarrow a - b \in \mathbb{Q}.$$
   (a) Show that R is an equivalence relation.
   (b) Give an example of one of the equivalence classes in A/R.
   (c) Prove that there are infinitely many equivalence classes in A/R.