Homework 3 - Math 300 D - Autumn 2014 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 3.1, and 3.2.

1. Let $a$ and $b$ be negative real numbers. Prove that if $a < b$ then $a^2 > b^2$.
2. Let $a, b$ and $c$ be positive integers. Prove that if $a|b$ and $b|c$, then $a|c$.
3. Let $a, b$, and $c$ be integers, $c \neq 0$. If $ac|bc$, then $a|b$.
4. One fact we use all the time when writing proofs is that, if $A \to B$ and $B \to C$, then $A \to C$. Prove this is valid by showing that

$$((A \to B) \wedge (B \to C)) \to (A \to C)$$

   is a tautology. Do this by using applicable laws, or a truth table, to show that this is equivalent to a statement which we know is a tautology.

5. Now that we know the irrational numbers exist, we should prove a few facts about them.

   You can use the following useful facts in your proofs. You do not have to prove them.

   Fact 1: The sum of rational numbers x=a/b and y=c/d is (ad+bc)/(bd).

   Fact 2: If a is rational, then -a is rational; if a is irrational, then -a is irrational.

   Prove the following theorems:

   (a) The sum of two rational numbers is a rational number.
   (b) The sum of a rational number and an irrational number is an irrational number.
   (c) The product of an irrational number and a non-zero rational number is an irrational number.
   (d) The sum of two irrational numbers may be a rational number.