Homework 2 - Math 300 D Autumn 2014 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 1.4, 1.5.

1. Verify each of the following set identities by showing that the statement "x is in the left-hand set" is equivalent to the statement "x is in the right-hand set". Justify each step.

   (a) $A \setminus (A \cap B) = A \setminus B$
   (b) $(A \cap B) \setminus C = (A \setminus C) \cap B$
   (c) $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$
   (d) $C \setminus (A \cup B) = (C \setminus A) \setminus B$
   (e) $(A \cap B) \setminus A = \emptyset$

2. Show the following without truth tables (i.e., use DeMorgan's law, associative law, etc.) Give justification for each step.

   (a) $P \Leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.
   (b) $(P \rightarrow Q) \wedge P$ is equivalent to $(P \wedge Q)$.
   (c) $(P \rightarrow Q) \wedge (P \rightarrow R)$ is equivalent to $P \rightarrow (Q \wedge R)$.
   (d) $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology.

3. Find a formula involving only $\neg$ and $\wedge$ that is equivalent to $P \Leftrightarrow Q$, and then find one involving only $\neg$ and $\rightarrow$ that is equivalent to $P \Leftrightarrow Q$.

4. Write useful contrapositives of the following sentences. Express the contrapositives as sentences, not as symbolic expressions.

   (a) If x and y are real numbers, then $x + y$ is a real number.
   (b) If x and y are integers, and at least one of them is even, then xy is even.
   (c) If you earned at least 90% in my class, then you got an A.
   (d) If it rains or snows, then I will go for a walk but I will not ride my bike.

5. Can we "distribute" with $\rightarrow$ and $\leftrightarrow$? That is, is

$$(P \rightarrow (Q \vee R)) \leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$$

   always true? What about

$$(P \rightarrow (Q \wedge R)) \leftrightarrow (P \rightarrow Q) \wedge (P \rightarrow R)$$

$$(P \leftrightarrow (Q \vee R)) \leftrightarrow (P \leftrightarrow Q) \vee (P \leftrightarrow R)$$

   and

$$(P \leftrightarrow (Q \wedge R)) \leftrightarrow (P \leftrightarrow Q) \wedge (P \leftrightarrow R)?$$

   Use truth tables or other means to show that each of these is valid or invalid.

6. Use truth tables to decide whether the following arguments are valid. Explain your conclusion.

   (a) It will rain or it will snow. If it snows, then I will go skiing. If it rains, then I will not go skiing. Therefore, it will not both rain and snow.
   (b) I will get a flat tire if and only if I ride my bike over glass and my tires are worn. My tires are not worn. Therefore I will get a flat tire if and only if I ride my bike over glass.
   (c) Angela or Boris has a toothache. Boris or Carla has a toothache, but they do not both have a toothache. Angela and Carla do not both have a toothache. Therefore, Angela has a toothache.