

Homework 1 - Math 300 D Autumn 2014 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 1.1, 1.2, 1.3.

1. Introduce variables and express the following sentences symbolically.

- (a) I will go to London or Paris, but not both.
- (b) Either Dave, Jing or Maria is lying, or they are all lying.
- (c) Peter and Xia are not both over two meters tall.
- (d) Peter and Xia are both not over two meters tall.
- (e) Olga is rich and famous, or she is not rich.
- (f) There is no cake left and I am hungry, or there is cake left but I am not hungry.

2. Write grammatical english sentences with the following structures.

- (a) $(P \wedge Q) \vee \neg P$
- (b) $(A \vee B) \wedge (\neg A \wedge B)$
- (c) $\neg(D \wedge F) \vee F$
- (d) $\neg(G \vee H) \wedge G$

3. Make truth tables of each of the following.

- (a) $P \wedge \neg Q$
- (b) $(P \wedge Q) \vee \neg P$
- (c) $(P \vee Q) \wedge (\neg P \wedge Q)$
- (d) $(P \vee Q) \wedge (P \vee R)$

4. Create truth tables for each of the following. Are any of them equivalent?

- (a) $A \wedge \neg B$
- (b) $\neg(A \vee \neg B)$
- (c) $(A \wedge B) \vee (\neg A \wedge B)$
- (d) $\neg A \vee (A \wedge \neg B)$
- (e) $(A \vee B) \vee (A \wedge \neg B)$

5. Simplify the following expressions.

- (a) $(P \vee (\neg P \wedge P)) \wedge \neg P$
- (b) $(P \wedge (Q \wedge R)) \vee (P \wedge R)$
- (c) $\neg(P \wedge (\neg P \wedge Q))$
- (d) $\neg(P \wedge \neg Q) \vee (P \wedge Q)$
- (e) $(P \vee Q) \wedge (P \wedge \neg Q)$
- (f) $\neg(\neg P \wedge Q) \wedge (\neg P \wedge Q)$
- (g) $(\neg Q \wedge (P \vee R)) \vee (P \wedge Q)$

6. Write the truth set of each of the following statements. Be as explicit as you can, and give complete justification for your answers (note: you may need to use calculus).

(a) n is an integer and $n^2 < 5$

(b) n is a positive integer and n is divisible by 2 or 5

(c) x is a real number and $\sin^2 x = 1$

(d) x is a real number and $\sin x = 0$ and $\sin 2x = 1$

(e) x is a real number and $x^2 - 6x + 1 = 0$

(f) x is a real number and $x^4 - x^3 + \frac{27}{256} = 0$

7. Evaluate each of the following arguments with truth tables, and conclude whether they are valid or not.

(a) • Andy and Bin are both tall, or neither of them is.

• Either Andy is tall or Bin is.

• Bin is tall.

(b) • Alex is friendly, and Bob or Clara is friendly.

• Bob is friendly, and Alex or Clara is friendly.

• Therefore, Clara is friendly.