Summary for Midterm Two - Math 120

Here are some thoughts I was having while considering what to put on the second midterm. The core of your studying should be the assigned homework problems: make sure you really understand those well before moving on to other things (like the old midterms on the test archive).

- Chapter 8 - Composition
  - In addition to combining two functions into a new function via arithmetic (the way we can combine two numbers into a new number), we can also combine two functions via *composition*.
  - You should understand what $f(g(x))$ means, and how to express a rule for $f(g(x))$ given rules for $f(x)$ and $g(x)$.
  - I especially like problems 8.3, 8.4, 8.5.
- Chapter 9 - Three Construction Tools
  - You should understand horizontal and vertical **shifting**, and horizontal and vertical **scaling** (aka dilating)
  - You should understand how to derive the graph of $g(x) = af(bx + c) + d$ from the graph of $f(x)$ (see, e.g., problem 9.2)
  - I especially like problem 9.2, 9.3, and 9.7
- Chapter 10 - Arithmetic
  - This is a very short chapter. An important topic in this chapter is **step functions**, which are a nice example of multipart functions.
  - You should understand how to graph functions built up from the unit step function (see problem 10.8)
  - You should be able to combine multipart functions and come up with the rule for the new function.
  - I really like problem 10.4, 10.5 and 10.8.
- Chapter 11 - Inverse Functions
  - Another very short chapter.
  - You should understand what an **inverse function** is, what conditions a function must satisfy in order to have an inverse (do all functions have inverses? can you tell if a function has an inverse by looking at its graph?), and how to find the inverse of a given function

- You should understand what a **one-to-one function** is, and what is special about the graph of a one-to-one function
- I like problem 11.7 and 11.8.

- Chapter 12 - Rational Functions
  - A very important chapter. We spent two days in lecture on this instead of the usual one.
  - You should be able to find the **asymptotes** (horizontal and vertical) of a **linear-to-linear rational function**, and be able to sketch the graph of a rational function like those in problem 12.1(a) or (b).
  - You should be able to model with **linear-to-linear rational functions**. This comes down to finding a rational function of the form

    $$f(x) = \frac{ax + b}{x + c}$$

    whose graph
    1. passes through three given points
       or
    2. has a given asymptote and passes through two given points
       or
    3. has two given asymptotes and passes throuh one given point

    You will need to translate the language of the modeling problem. Take a look at old midterm 2 exams from the archive for examples to work on.

    Pay particularly close attention to the words "linear-to-linear".

    Note that a linear-to-linear function is not a **linear function**.
  - I especially like problems 12.1, 12.7, 12.8, 12.9, 12.11, 12.12.

- Chapter 13 - Measuring an Angle
  - You should understand how to convert between **degrees** and **radians**
  - You should understand and be able to use the relationships between **radii**, **angle**, **arc length** and **area**
  - I like problems 13.8 and 13.9.

- Chapter 14 - Measuring Circular Motion
  - You should understand the various measures of **angular speed** (aka **angular velocity**), like rpm, radians per second, or degrees per hour

- You should understand the relationship between **radius**, **angular speed** and **linear speed**
- You should know how to solve a belt-and-pulley problem (e.g., the bicycle example from lecture, example 14.4.1, problems 14.3, 14.9 and 14.11)
- I like problems 14.5 and 14.7.

- Chapter 15 - The Circular Functions
  - This chapter introduces the **trigonometric functions**.
  - You should be able to solve problems using the idea of trigonometric functions as ratios of sides of right triangles (e.g., problems 15.4, 15.7, 15.8) and some algebra
  - You should understand the definitions of $\sin x$ and $\cos x$ using the **unit circle**; you should be able to determine certain simple properties of the functions $\sin x$ and $\cos x$ from this definition (e.g., the range, the domain, the graph, the values at certain value of x, like $x = 5\pi/2$)
  - You should be able to determine the location of an object moving circularly given information about its speed and starting location (e.g., problems 15.2, 15.5, 15.9, 15.15)

- Chapter 16 - Trigonometric Functions
  - This is a short chapter which adds some final touches to our knowledge of the functions $\sin x$ and $\cos x$ and related functions.
  - I like problems 16.3 and 16.4.