Math 120 Final Review Problems

1. A wheel of radius 5 inches is spinning counterclockwise at 260 rpm. The wheel is horizontal, like a potter's wheel, or a carousel. A speck of dirt on the edge of the wheel flies off the wheel 0.01 seconds after it is at the easternmost point of the wheel. The speck then travels along a path that is tangent to the wheel. Assuming the dirt continues in a straight line, where does the speck of dirt hit a north-south wall that is located 9 inches west of the wheel's center? (answer: 50.827 inches north of the wheels center)

2. A circular swimming pool has a diameter of 18 feet. There is a tree 11 feet north and 5 feet east of the center of the pool. There is a ladder at the southermost point of the pool. Callie wants to play water polo with her friends and decides to string a net between the ladder and the tree. How long is the part of the net that is over the swimming pool? (answer: $\dfrac{72}{\sqrt{17}}$)

3. A six foot tall baseball shortstop throws a ball to the first baseman. The table below gives some information about the position of the ball in flight.

distance from the shortstop (in feet)	0	25	50
Distance off the ground (in feet)	6	9.25	10.5

   (a) Find a quadratic function, $f(x)$, that represents the altitude of the ball as a function of its distance x from the shortstop. (answer: $f(x) = -\dfrac{1}{625}x^2 + \dfrac{17}{100}x + 6$)

   (b) What is the highest altitude reached by the ball? (answer: $\dfrac{673}{64}$)

   (c) If the first baseman is also 6 feet tall, how far away from the shortstop is he when he catches the ball? (answer: $\dfrac{425}{4}$)

4. Give the zeros and asymptotes (if any exist) of the following rational function:
$$y = \frac{2x^2 - 2}{4x^2 + 4x - 80}.$$
   (answer: zeros: $x = 1, x = -1$; vertical asymptotes: $x = -5, x = 4$; horizontal asymptote: $y = \frac{1}{2}$)

5. Let h be a constant. For the function
$$f(x) = -2x - 4$$
   calculate
$$\frac{f(x+h) - f(x)}{h}$$
   and simplify so that plugging in $h = 0$ is defined (answer: -2).

6. The light in a lighthouse rotates at the rate of 12 RPM. The beam of the light reaches 7 miles into the surrounding fog. Give a formula for the area $A(t)$ swept out by the light after t seconds. (answer: $A(t) = \dfrac{49}{5}\pi t$)

7. Two houses are located 8 miles apart. Each house is 1 mile from a road. Sally can jog 8 miles/hour along the road, but only 3 miles/hour in the dirt (off-road). Because of a river directly between the houses, it is necessary for her to jog from one house to the other by running in the dirt to the road, then along the road, and then in the dirt to the other house, as shown in the figure. Express the time T to get from one house to the other as a function of the angle θ shown in the figure. (answer: $T(\theta) = \dfrac{2}{3\sin\theta} + 1 - \dfrac{1}{4\tan\theta}$)
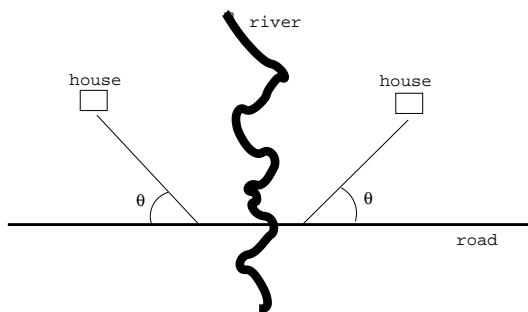
8. Suppose you are riding a bicycle along the Burke-Gilman Trail at a constant pedaling rate of 2 revolutions per second. The wheels on your bike are 26 inches in diameter. The rear sprocket has a radius of 3 inches. You have two front sprockets, one with a radius of 4 inches, the other with a radius of 5 inches. How much faster would you be riding if you were using the 5 inch sprocket compared to the 4 inch sprocket? (answer: 4.537856 feet per second)
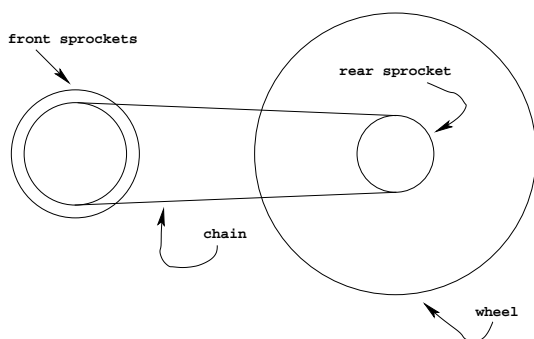
9. A function $f(x)$ is called *odd* if $f(-x) = -f(x)$ for all x in the domain of the function. A function is *even* if $f(-x) = f(x)$ for all x in the domain of the function. Most functions are neither even nor odd; for instance, $f(x) = x^2 + 3x + 2$ is neither even nor odd. However, all functions (whose domains are all real numbers) can be written as the sum of an even function and an odd function. Here's how. Suppose $f(x)$ is defined for all x. Then let $e(x) = \dfrac{f(x) + f(-x)}{2}$, and $o(x) = \dfrac{f(x) - f(-x)}{2}$. Now complete the following exercises:

   (a) Show that f(x) = e(x) + o(x).

   (b) Show that e(x) is an even function and o(x) is an odd function.

   (c) Let $f(x) = x^2 + 3x + 2$. Find $e(x)$ and $o(x)$ for this function, and convince yourself that $f(x) = e(x) + o(x)$, and that $e(x)$ is even, and $o(x)$ is odd. (partial ans.: $e(x) = x^2 + 2, o(x) = 3x$)

   (d) Let $f(x) = e^x$, and use the method about to express it as the sum of an even function and an odd function. What are those functions? (Hint: consider problem 20.7 from the text)

10. Boris and Alice have teamed up and purchased a peach orchard and an apple orchard. In each orchard, the number of kilograms of fruit each tree produces is a linear function of the number of trees in that orchard.

   If there are 100 trees in the peach orchard, each tree will produce 25 kg of fruit. If there are 110 trees, each tree will produce 23 kg of fruit.

   If there are 100 trees in the apple orchard, each tree will produce 36 kg of fruit. For each addition of 12 trees, there is a decrease of 4 kg in the amount of fruit each tree produces.

   For the purposes of this problem, allow the existence of fractions of trees.

   (a) Boris wants the number of trees to be the same in each orchard, and the *total* amount of fruit produced in each orchard to be equal. How many trees should there be in each orchard to make this happen? (ans. 182.5 trees)

(b) Alice wants the number of trees in each orchard to be equal, but wants the total amount of fruit (apples plus peaches) produced to be maximized. How many trees should there be to make this happen? (ans.: 107.1875 trees)

11. Suppose the daily maximum temperature in a city is a sinusoidal function of the day of the year. Suppose the maximum of this function, 102 °F occurs once per year, on the 210th day of the year, and the minimum of this function is 4°F. For convenience, assume each year has 366 days. On what days of the year is the daily maximum about 75°F ? (ans.: the 145th and the 274th)

12. According to Newton's Law of Cooling, the temperature of an object changes at a rate proportional to the difference between the temperature of the object and the temperature of its surroundings. With calculus, we can show that the difference between the temperature of an object and its surroundings changes according to an exponential function. Suppose the temperature of a cup of coffee is given by

$$C(t) = 20 + Ke^{rt}$$

where t is in minutes, K and r are constants, and the temperature is degrees Celsius. Initially the coffee is $80°$. After 1 minute, the coffee is $60°$.

(a) What is the temperature of the coffee after 2 minutes? (ans.: $46.6667°$)

(b) When will the temperature of the coffee be $30°$? (ans.: after 4.41902 minutes)

(c) After a long time, what will the temperature of the coffee be? (ans.: essentially $20°$)