A compostion example

Let

$$f(x) = |x|$$

and let

$$g(x) = f(f(x) - 2)$$

What is the multipart rule for $g(x)$?

Using the multipart rule for the absolute value function:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

we can write

$$g(x) = ||x| - 2| = \begin{cases} |x| - 2 & \text{if } |x| - 2 \geq 0 \\ -|x| + 2 & \text{if } |x| - 2 < 0 \end{cases}$$

$$= \begin{cases} |x| - 2 & \text{if } |x| \geq 2 \\ -|x| + 2 & \text{if } |x| < 2 \end{cases} = \begin{cases} |x| - 2 & \text{if } x \geq 2 \text{ or } x \leq -2 \\ -|x| + 2 & \text{if } -2 < x < 2 \end{cases} = \begin{cases} x - 2 & \text{if } x \geq 2 \\ -x - 2 & \text{if } x \leq -2 \\ -x + 2 & \text{if } 0 \leq x < 2 \\ x + 2 & \text{if } -2 < x < 0 \end{cases}$$

Putting these rules in order we have, finally,

$$g(x) = \begin{cases} -x - 2 & \text{if } x \leq -2 \\ x + 2 & \text{if } -2 < x < 0 \\ -x + 2 & \text{if } 0 \leq x < 2 \\ x - 2 & \text{if } x \geq 2 \end{cases}$$