

## RULES:

- (1) *The only electronic device allowed during the quiz is a **TI 30X IIS** calculator.*
- (2) *TURN OFF AND PUT AWAY YOUR PHONE!* If the quiz proctor sees or hears your phone *at any time on a quiz day*, your quiz will not be graded and you will receive a grade of zero for the quiz.
- (3) Leave your quiz face up on your desk until the quiz proctor tells everyone to begin.
- (4) When the quiz proctor announces the end of the quiz, you have one minute to give your quiz to the proctor. If the proctor does not have your quiz within one minute after the end of the quiz, your quiz will not be graded.
- (5) At any point before or during the quiz, the quiz proctor may request that you change your seat. Please do so promptly.
- (6) Close all purses and backpacks and place them under your chair with your phone inside and off.
- (7) Only your quiz, writing implements and calculator (if you use one) may be on your desk during the quiz.
- (8) No supplemental material, such as notes, are allowed during the quiz.
- (9) The quiz proctor may ask you to present a photo ID at any point during the quiz. If you do not have one, then you will be asked to surrender your quiz, and the quiz will not be graded.

### Problem

Score

1

2

Total

1. [a](20 points) Define the following norms on  $\mathbb{R}^n$ .

(i)  $\|x\|_2 =$

(ii)  $\|x\|_1 =$

(iii)  $\|x\|_\infty =$

[b](20 points) Let  $A$  be a linear transformation from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ . What are the 4 fundamental subspaces associated with  $A$  and how are they related?

[c](20 points) What is the Gaussian-Jordan elimination matrix that transforms the vector

$$\begin{pmatrix} 1 \\ 2 \\ 4 \\ 10 \end{pmatrix} \quad \text{into the vector} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

2. [a] (20 points) Set  $\mathbb{R}_+^n = \{x \in \mathbb{R}^n \mid 0 \leq x_i, i = 1, 2, \dots, n\}$  and  $\mathbb{R}_{++}^n = \{x \in \mathbb{R}^n \mid 0 < x_i, i = 1, 2, \dots, n\}$ . Compute  $\nabla f(x)$  and  $\nabla^2 f(x)$  at a point  $x \in \mathbb{R}_{++}^3$  for the function  $f: \mathbb{R}_{++}^3 \rightarrow \mathbb{R}$  given by

$$f(x) = \frac{1}{2} \|x\|_2^2 - \log(x_1 x_2 x_3),$$

where  $x = (x_1, x_2, x_3) \in \mathbb{R}_{++}^3$ , and  $f$  is undefined otherwise.

[b] (20 points) Let

$$A := \begin{bmatrix} 2 & -1 & 1 \\ 0 & 7 & -2 \\ 4 & 0 & -3 \\ -1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad b := \begin{pmatrix} 2 \\ 1 \\ 4 \\ -1 \end{pmatrix},$$

and define  $f: \mathbb{R}^3 \rightarrow \mathbb{R}$  by  $f(x) := \frac{1}{2} \|Ax - b\|_2^2$ . Compute  $\nabla f(0)$  and  $\nabla^2 f(0)$ .