GUIDE FOR MIDTERM EXAM 1

MATH 208A **Exam date: October 20, 2021**

The exam will have 4 questions with each question being multipart. A loose description of the content of each question is given below along with a sample question for illustration and practice. The rules for the exam are listed at the end of this guide.

Question 1: This question is based on your understanding of the vocabulary words for sections 1.1, 1.2, 2.1, 2.2, 2.3, and 3.1. See the Weekly Overviews on the course website for a complete list of the vocabulary words.

(Sample Q1) Let $v^1, \dots, v^k \in \mathbb{R}^n$ and let $V \in \mathbb{R}^{n\times k}$ be the matrix having columns $v^1, \dots, v^k$. For each item in the left hand column circle the letter of the entries in the right hand column that are equal to it. Note that a given entry in the left hand column may equal several entries on the right or may not equal any entry on the right. One point for each correct answer and minus one point for each incorrect answer. The total score will be the maximum of the sum of the points and zero, but no greater than 5 points.

Ran (V) a b c d e f (a) $\left\{\alpha_1 v^1 + \alpha_2 v^2 + \cdots + \alpha_k v^k \,:\, \alpha_1, \alpha_2, \dots, \alpha_k \in \mathbb{R}\right\}$

Nul (V) a b c d e f (b) $\left\{(\alpha_1, \alpha_2, \dots, \alpha_k) \,:\, \alpha_1 v^1 + \alpha_2 v^2 + \cdots + \alpha_k v^k = 0,\ \alpha_1, \alpha_2, \dots, \alpha_k \in \mathbb{R}\right\}$

(c) $\left\{y \in \mathbb{R}^n \,:\, \exists\, x \in \mathbb{R}^k \text{ such that } y = Vx,\ y_1 v^1 + y_2 v^2 + \cdots + y_k v^k = 0\right\}$

(d) $\left\{x \in \mathbb{R}^k \,:\, Vx = 0\right\}$

(e) $\left\{y \in \mathbb{R}^n \,:\, \exists\, x \in \mathbb{R}^k \text{ such that } y = Vx\right\}$

(f) Span $\left[v^1, \dots, v^k\right]$

Question 2: This question addresses properties of matrices $A \in \mathbb{R}^{n\times n}$ specifically concerning their echelon form, reduced echelon form, and the associated linear transformation $T_A(x) = Ax$. In this regard, the pivot structure of the echelon form and reduced echelon form are important as well as the implications for when T_A is one-to-one, onto, neither, or both.

Sample Q2:

(a) Let $A \in \mathbb{R}^{n\times m}$ and let $B \in \mathbb{R}^{n\times m}$ be a matrix obtained by transforming A into echelon form.
  (i) What must be true about B for Ran $(A) = \mathbb{R}^n$?
  (ii) What must be true about B for Nul $(A) = \{0\}$?

(b) Let $A = \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 2 & 1 & -1 & 4 & 3 \\ 3 & 1 & -7 & 5 & 5 \end{bmatrix}$.
  (i) Is A one-to-one?
  (ii) Is A onto?

Note: Simply answering "yes" or "no" to this question gives zero points. You must validate your answer with an appropriate computation where all of your work is shown.

Question 3: This question is strictly computational. You will be given an explicit system of linear equations and asked to give a complete description of its set of solutions in vector form, i.e., a particular solution plus a linear combination of vectors solving the associated homogeneous equations. On this problem you should organize your work so that you can easily find whatever errors you make during the computations and correct them. Obviously you will be expected to check the correctness of the solution you give.

Sample Q3: If the following linear system has a nonempty solution set, write the set of solutions in vector form.

$$\begin{array}{rcrcrcrcrcr} x_1 & + & & - & x_3 & + & 2x_4 & + & x_5 & = & 1 \\ 2x_1 & + & x_2 & - & x_3 & + & 4x_4 & + & 3x_5 & = & 1 \\ 3x_1 & + & x_2 & - & 7x_3 & + & 5x_4 & + & 5x_5 & = & 1 \end{array}$$

Note: You must show all of your work. Most of the points are given for the work, not just the answer. Points are also given for checking your answer. That is, even if you compute the correct answer but do not check your answer, you will not be given full points.

Question 4: This question concerns the linear span of a collection of vectors, and their linear independence and/or dependence.

Sample Q4: Let

$$u^1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \quad u^2 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \quad \text{and} \quad u^3 = \begin{pmatrix} z_1 \\ -4 \\ z_2 \end{pmatrix}.$$

(a) Find all values z_1 and z_2 such that u^1, u^2, and u^3 do **not** span $\mathbb{R}^3$.
(b) Find all values z_1 and z_2 such that u^1, u^2, and u^3 are linearly independent.

The rules for the exam are listed below.

(1) Leave your exam face up on your desk until the exam proctor tells everyone to begin.
(2) *TURN OFF YOUR PHONE!* Close all purses and backpacks and place them under your chair with your phone inside and **off**. If the proctor sees or hears a phone during the exam, you will be asked to surrender either the phone or your exam.
(3) You may use a Texas Instruments TI-30X IIS. *NO OTHER ELECTRONIC DEVICES ARE ALLOWED!*
(4) One handwritten 8.5 by 11 sheet of notes is allowed. 2-sided is OK.
(5) Only your exam, notes, calculator, and your writing implements may be on your desk during the exam.
(6) There are 4 problems. When you are told to begin the exam you must first make sure you have all of the problems. If you do not have them all, then request a new exam.
(7) Show all of your work and follow the directions provided. Partial credit will be given for partial solutions.
(8) When the exam proctor announces the end of the exam, you have two minutes to give your exam to the proctor. If the proctor does not have your exam within two minutes after the end of the exam, your exam will not be graded.
(9) At any point before or during the exam, the exam proctor may request that you change your seat. Please do so promptly.
(10) You may be asked to present a photo ID (student id or valid drivers license) at any point during the exam. If you do not have one, then you will be asked to surrender your exam.