INTRODUCTION TO THE COURSE

Topology is sometimes described as the math that can't tell a coffee cup from a donut. The idea is that topology cares only about properties that are preserved by continuous deformation. By such a deformation, we move most of the mass of the donut to one side. Leaving the "small" part of the donut to be the handle of the coffee cup, we flatten the fat part into a disc and then shape the disc into the cup part which holds the coffee.

To make these intuitive descriptions into mathematical arguments, we need mathematically precise versions of the definitions and techniques used, An even bigger challenge is presented by comparing the [surface of a] donut and a sphere. It seems obvious that to make a sphere into a donut, we'd have to rip a hole in it, and that isn't "continuous deformation." But just trying and failing to find a continuous deformation between the two is not a proof. We must show there isn't *any* possible way to find one. Proving this requires a topological point of view, a new conceptual framework. In this topological viewpoint, familiar ideas like continuity, limits, and what it means for points to be "near" each other are abstracted until they may seem unrecognizable. The high level of abstraction is a major reason many students find Math 441 very challenging.

A second reason is the emphasis on proof. In this course, "problem" will usually mean "proof." Past experience with Math 441 has shown that the usual "lecture, then homework" model isn't effective for helping all students develop a new conceptual framework and proof creation skills. And recent education research has shown that there is a better way to organize a course. Students should get an introduction to basic information and do some initial work, before class. The advantages of classtime — the instructor as an information source and interacting with other students — are best used for checking and extending the work done before class. After class, additional homework solidifies knowledge and skills.

But doesn't the most effective course organization vary for different students? It may seem like the new plan might not be the best for the students who've always done well in traditional lecture courses. Or it might not work well for students at the other extreme, those who are struggling the most with the course. Surprisingly, the research shows that when performance data is split into groups like this, usually all groups do better with the new plan than in traditional lecture courses, no group ever does worse, and most or all groups in most classes do significantly better. (Some of the research is specifically on college math classes, including post-calculus, and lots has been done on college science classes. If you are interested in more details, see links to the research on the course web and Canvas site.)

COURSE ORGANIZATION

For almost every class, you will prepare with reading and Warmup Problems (WP). A written Reading Response (RR) will be due 24 hours before the start of class. (Details on content and turn-in for this and other assignments given elsewhere.) You will turn in a copy of your advance work on the WP by the start of class.

Classtime will usually start with a discussion of questions raised in the reading responses. Then you will work in groups to check the WP, asking the instructor(s) for help. After your

group has agreed on a solution, you may be called on to post it on the board for whole class discussion.

Followup Problems (FP) will be assigned on the material from each class, and will be due once a week. You may work with others in the class to figure out these problems, but do not look at anyone else's final written solution. The work you had in must be your own thoughts in your own words.

A word of warning. Many previous 441 students who used the internet to help with homework became dependent on "internet-assisted" proof writing. This resulted in artificially high homework scores and miserably poor exam scores.

DO NOT LOOK FOR MODEL PROBLEM SOLUTIONS ON THE INTERNET!

The best idea is to avoid all use of the internet for this course except for the course website and Canvas site.

DOES THIS REALLY WORK?

In 2014 I taught this course as a tradition lecture course. The median on the midterm was about 50%. Several students, some with homework averages above 85%, earned midterm scores between 3% and 20% on that midterm. Last fall, I taught using the course organization described above. The midterm had exactly the same two first questions as in 2014, and the other two questions were similar to the other two on the 2014 exam in type and difficulty. The median was just over 79%, and the lowest score was just over 38%. Unfortunately this big improvement did not continue through the quarter. In 2014, some students dropped the course after the midterm and the rest did somewhat better on the final. No one in 2018 class dropped after the midterm, and the average on the final was lower than the midterm, with no significant difference between the two years' final exam performances. We should all keep this in mind in the latter half of the course. The material in the second half is very challenging. I will look for ways to improve how we work on it, and you should be sure not to slack off, even if you did well on the midterm.

Student opinion on the course organization was somewhat mixed, but largely positive. A midquarter survey asked which types of assignments had been useful for learning topology (multiple answers allowed). For the RR, WP, and FP assignments, 72%, 86%, and 83% of the class, respectively, said they had been helpful. In commments, the biggest complaint was the number of assignments, and how much time they took. This was one reason to switch this year to longer classes twice a week: You will only have two WP assignments a week, compared to three a week for last year's class.

In evaluations at the end of the quarter, comments were still more positive than negative. While some students said they would prefer traditional lectures for some or all of classtime, one of those wrote,"at times I wonder if time would be more well spent if there was more lecturing. Although, perhaps I am retaining the info more by discussion, it is hard to tell."

One student told me at graduation in June, "I liked how we did the course. I felt like I was learning topology instead of just getting through the homework." Here are some other student comments from the miquarter survey and course evaluations at the end of the quarter. Samples of both positive and negative comments are included.

"The active learning was, I have to say, quite incredible, despite my misgivings."

"I respect the active learning pedagogy that was tried this quarter, but I just don't think it worked too well with me. ... I'm sure the style of teaching worked for some (and maybe even most) people though, so this is just my personal opinion."

"The WP were good for focusing my studies and being able to ask specific questions on the RR was helpful."

"I don't feel like I'm getting that much from the reading responses. I don't know what I don't understand until someone makes me do a problem with it, which is why I think the warm up problems make a big difference." [JMA remark: This comment says that the reading and problems together were useful, even though the student seems to think that the problems were useful while the reading was not.]

"I only have complaints because I feel like I cannot do my work on my own schedule. I guess the RR and WP must be very helpful because even though they can be annoying I cannot deny that they help me learn the material well."

"I often find that I read the problems, exclaim to myself, 'This is easy!', type up a quick solution, and neglect to give them another thought until class. Once we begin the group discussion, however, I realize that my arguments more often than not contain a serious flaw." [JMA remark: This is ideal: working on the problem before class prepares you to understand and appreciate the tricky points much better than if you are just told the full solution, without trying to do it yourself. This is true even if, unlike this student, you don't produce a complete solution before discussion.]

"I personally did not find group discussions to be that helpful. It was hard to admit when I didn't understand something and sometimes other students made it seem like it was 'easy'."

"The follow up problems helped me understand the content deeply, the warm up problems helped me explore the content, and the reading responses helped me reflect on the content. All three were necessary in my exploration."