## 14.5: The Chain Rule

• Using all versions of the chain rule. Here is one case:  $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$ 

# 14.6: Directional Derivatives and Gradients

- Computing the gradient. Here is the 3D version:  $f(x, y, z) = \langle f_x(x, y, z), f_y(x, y, z), f_z(x, y, z) \rangle$ .
- Computing directional derivatives. For a unit direction vector  $\mathbf{u}$ ,  $D_{\mathbf{u}}f(x, y, z) = f \cdot \mathbf{u}$ , which gives a number representing the slope on the surface if you are starting at the point (x, y, z) and looking in the direction  $\mathbf{u}$ .
- Understand the significance of the gradient. The gradient points in the direction of greatest increase on the surface and the magnitude of the gradient is the largest directional derivative at that point (steepest slope).

### 16.1: Introduction to Vector Field

• Know the basic terminology and be able to work with vector fields.

### **16.2:** Line Integrals

- Compute a line integral for a scalar function with respect to arc length:  $\int_C f(x,y) \, ds$  or  $\int_C f(x,y,z) \, ds$ , where  $ds = |r'(t)| dt = \sqrt{(x'(t))^2 + (y'(t))^2} dt$ .
- Computing line integrals over a vector field (and examples directly with respect to x and y).  $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P(x, y) dx + Q(x, y) dy \text{ or equivalently in 3D.}$
- To compute either case, you need to know how to parameterize curves.

### 16.3: Conservative Vector Fields

- Be able to test if a vector field is conservative.  $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$
- Finding the potential function for a conservative vector field. Integrate first component with respect to x. Then anything you haven't yet found in the second component, integrate with respect to y. Then anything you haven't yet found in the third component, integrate with respect to z.
- Using the potential function evaluate a line integral.  $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C f \cdot d\mathbf{r} = f(\mathbf{r}(b)) f(\mathbf{r}(a)).$
- Some set terminology basics.

### 16.4: Green's Theorem

• Be able to use Green's Theorem. 
$$\oint_C P(x,y)dx + Q(x,y)dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA$$

#### 16.5: Curl and Div

• Be able compute curl and div.

$$\operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P(x, y, z) & Q(x, y, z) & R(x, y, z) \end{vmatrix} \text{ and } \operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

- Know the input and output of each operation.
- Be able to check if a vector field is conservative (if curl  $\mathbf{F} = \langle 0, 0, 0 \rangle$ ).
- Be able to determine if a vector field is the curl of another vector field (if div  $\mathbf{F} = 0$ ).

#### **16.6:** Parameterizing Surfaces

- Be able to parameterize surfaces.
- You should also look at the examples from the text, examples from lecture, examples from my review sheet, and other examples from the problems in the book.