

Old midterm 2 answers

1. (a) $y = c_1 e^{4t} + c_2 t e^{4t}$
(b) $y = c_1 \cos(3t) + c_2 \sin(3t) + \frac{2}{3} t \sin(3t)$
2. $y_2 = t$
3. $x = (\pm)2$ ft
4. (a) $m = 1/400$

Object	Natural frequency
(b) A	$\sqrt{100} = 10$
B	$\sqrt{100/4} = 5$
C	$\sqrt{100/8} = 5/\sqrt{2}$

B by resonance (damping is small relative to k and m)

5. (a) $u(t) = e^{-2t} \left(-\frac{1}{4} \cos(2t) - \frac{1}{4} \sin(2t) \right)$
(b) The object first returns to equilibrium position after $3\pi/8$ sec.
(c) $t = -\frac{1}{2} \ln\left(\frac{\sqrt{8}}{120}\right)$

BONUS: $y = c_1 e^t + e^{-t/2} \left(c_2 \cos(\sqrt{3}t/2) + c_3 \sin(\sqrt{3}t/2) \right) - t^2$