Math 307, Midterm 2
Winter 2013

Name: ______________________________

Instructions.

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question. Come back to the question you left if you have time at the end.
- There are 5 questions on 6 pages. Make sure your exam is complete.
- You are allowed one double-sided sheet of notes in your own handwriting. You may not use someone else's note sheet.
- You may use a simple scientific calculator, but you don't need to. No fancy calculators or other electronic devices allowed. If you didn't bring a simple calculator, then just don't use a calculator.
- It's fine to leave your answers in exact form. But you should simplify the trig functions whose values you know from the unit circle (e.g., write "0" instead of "sin(0)"). You also should simplify expressions like $e^{-2\ln t}$.
- **Show your work**, unless instructed otherwise. If you need more space, use the back of the last page. If you still need more space, just raise your hand and I'll give you more paper to staple on the back of the test.
- Don't cheat. If you do, you will receive a zero on the exam and I will report you to the university.

Question	Points	Score
1	10	
2	10	
3	8	
4	14	
5	8	
Total:	50	

1. Find the general solution to each differential equation.

   (a) (4 points) $y'' + 2y = 5e^{-t}$

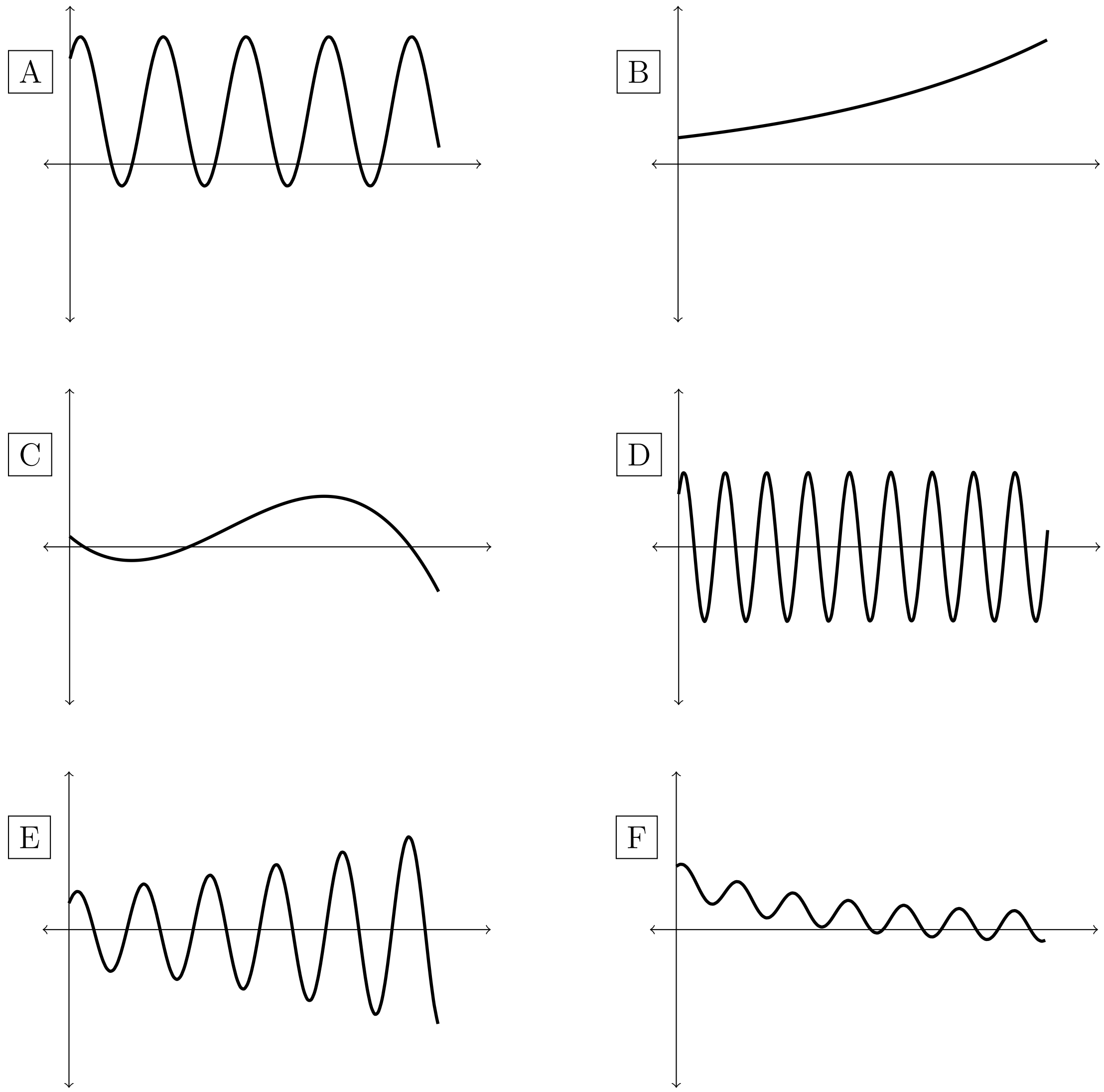
   (b) (6 points) $y'' + 9y = 12\sin(3t)$

2. (10 points) Consider the initial value problem

$$x^2y'' - 2y = 0, \qquad y(1) = 5, \qquad y'(1) = 1$$

for $x > 0$. I have already found one solution to the differential equation: $y = x^2$. (You don't need to check that this is a solution.) Solve the initial value problem.

3. (8 points) Below are six graphs of functions $y(t)$. The t and y axes are shown. The graphs are labeled A, B, C, D, E, F.



For each differential equation below, figure out what its solutions would look like. If one of the graphs above could be a solution, write its label in the blank space. If none of the graphs could possibly be a solution, write "NONE" (don't just leave it blank). Some graphs might not be used, and some might be repeated. No work necessary.

$y'' - 4y' + 4y = 0$ __________	$t^2y'' + 17t^2y = 0$ __________
$y'' + 4y = 0$ __________	$y'' + 4y' + 3y = 0$ __________
$y'' + 4y' + 5y = 0$ __________	$y'' - y = 0$ __________
$2y'' - 6y' + 4y = 0$ __________	$y'' - 2y' + 2y = 0$ __________

4. A spring has spring constant $k = 1$. An 9-kg mass is attached to the end of the spring. Let γ be the damping constant of the system.

   (a) (8 points) Suppose $\gamma = 10$. Zach pulls the mass down from equilibrium by $\frac{1}{9}$ m, and releases it at $t = 0$ with an initial **upward** velocity of 1 m/s. Write and solve an initial value problem that models the motion of the mass. There is no external force. (Be careful with your signs!)

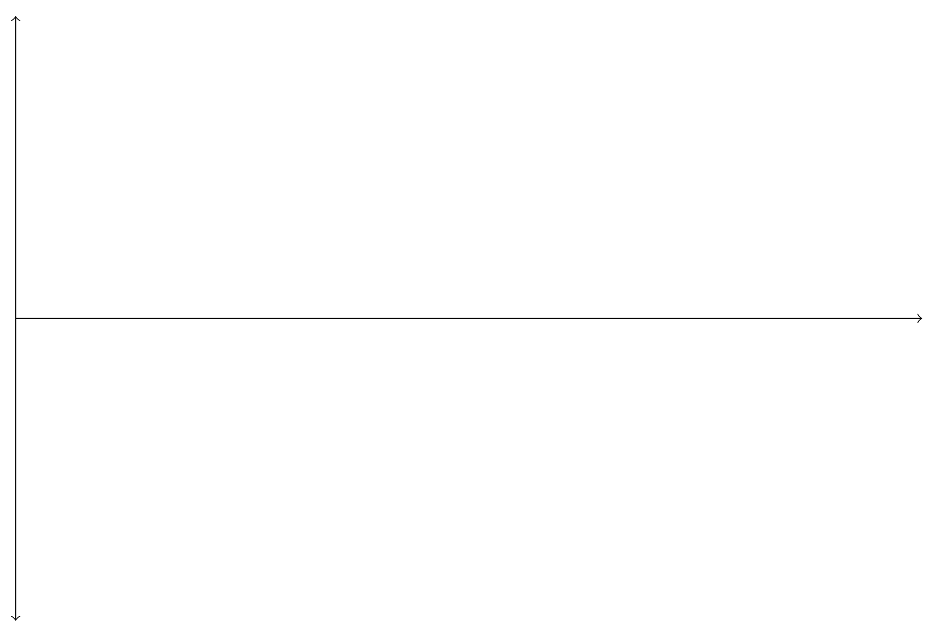
(Problem 4 continued...)

(b) (2 points) How many times does the mass pass through its equilibrium after $t = 0$? If it does so infinitely many times, say so. Show your work.

(c) (2 points) Now suppose the system is critically damped. Then what is γ?

(d) (2 points) In this part, you don't need to compute anything. Just draw a sketch, and tell me why you think your answer is right.

Suppose γ is equal to the number you found in part (c). If Zach pulls the mass down and releases it from rest, what happens? Sketch a graph of the motion $u(t)$, starting at $t = 0$.



5. I have four identical spring-mass systems. Each system has a mass of 2 kg, a spring constant of 18 N/m, and a damping constant of $\frac{1}{9}$ N·s/m (or kg/s).

   I also have four motors, which I attach to the four spring-mass systems. The motors apply to each system an external force of $2\cos(\omega t)$, where ω is given in the table below:

System:	System 1	System 2	System 3	System 4
ω:	$\omega = 2.95$	$\omega = 4.1$	$\omega = 8.85$	$\omega = 17.9$

   The masses begin to oscillate when I turn on the motors. After 20 seconds (long enough so that the motion is basically indistinguishable from the steady state response), I look at the systems.

   (a) (3 points) Which spring-mass system is moving with the greatest amplitude? Why? You don't need to show me calculations, but you must explain your answer clearly. No credit for a guess.

   (b) (2 points) For the system you found in part (a), tell me about the amplitude of its motion. How do you expect it to compare to 2, the amplitude of the forcing function? Why? No calculations necessary, but you must explain.

   (c) (3 points) Which system has the smallest amplitude? Again, explain your answer.