Math 307 - Spring 2016
Exam 2
May 18, 2016

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

- There are 5 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions.
DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
- You have 50 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 10 MINUTES PER PAGE!

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PAGE 4	10	
PAGE 5	10	
Total	50	

GOOD LUCK!

1. (10 pts) For both parts below, give the general solution:

   (a) $y'' + 2y' + y = 3t^2 - 1$

   (b) $y'' - 4y = 5 + 3e^{2t}$.

2. (10 pts) **For ALL parts**, assume the mass-spring system has a mass of $m = 2$ kg, a spring constant $k = 5$ N/m, and NO external forcing. Thus, $2u'' + \gamma u' + 5u = 0$.
Include UNITS in your final answers.

   (a) Assume NO damping and the initial conditions $u(0) = 0.5$ m and $u'(0) = 1$ m/s.
   Find the solution (find all constants).

   (b) Assume there is damping with $\gamma = 2$ N/(m/s). The solution exhibits vibrations (with decreasing amplitude). What is the quasi-period?

   (c) Give the smallest value of γ for which the solution will NOT exhibit vibrations.

3. (10 pts) **For ALL parts**, assume the mass-spring system has a mass of $m = 2$ kg, a spring constant $k = 5$ N/m, **and** an external forcing of the form $F(t) = F_0 \cos(\omega t)$ Newtons. Thus, $2u'' + \gamma u' + 5u = F_0 \cos(\omega t)$.

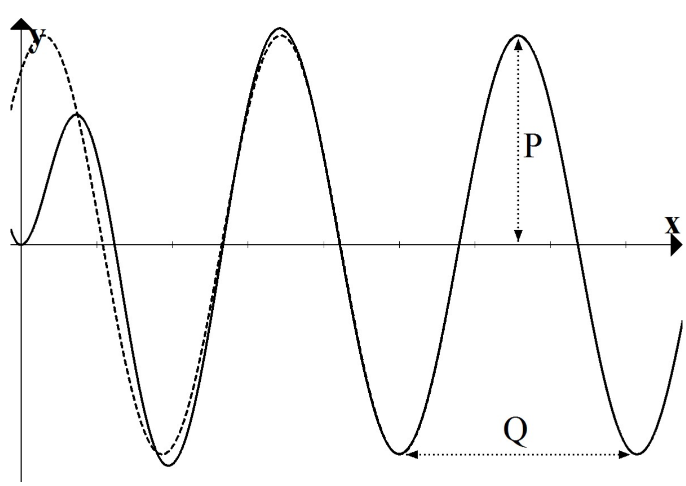
(a) Assume there is NO damping. What particular value of ω will lead to vibrations with increasing and unbounded amplitude?

(b) Assume there is damping with $\gamma = 2$ N/(m/s) and $u(0) = 0$ m and $u'(0) = 0$ m/s. Also assume $F(t) = 39 \cos(t)$ N. You are told the solution takes the form:

$$u(t) = c_1 e^{\lambda t} \cos(\mu t) + c_2 e^{\lambda t} \sin(\mu t) + 9 \cos(\omega t) + 6 \sin(\omega t).$$

- What are the values of λ, μ, ω, c_1, and c_2? (You only have to give units for μ and ω).

- The graph of the full solution (solid) and the steady state solution (dotted) are given below. Find the indicated lengths P and Q. (include units!)



4. (10 pts) Consider the model $u'' + 4u' + 3u = 0$ with $u(0) = 0.3$ m, $u'(0) = -1$ m/s.

(a) What can we say about this system? (**Circle one**):

Critically Damped OR Overdamped OR Exhibits Vibrations.

(b) Solve for $u(t)$. (find all constants)

(c) Find the one, and only, time the mass will be at the equilibrium position (*i.e.* when $u(t) = 0$).

5. (10 pts) (The two parts below are not related)

(a) A 3 kg object stretches a spring 10 cm beyond its natural length (and is at rest).
The damping force is 5 N when the upward velocity is 6 m/s. There is no external forcing.
Initially, the mass is pushed upward 5 cm and given an initial downward velocity of 20 cm/s.
Set up the differential equation AND initial conditions for the displacement $u(t)$.
Watch the units! (DO NOT SOLVE)

(b) The function $y_1(t) = t^2$ is one solution to the homogeneous equation $t^2y'' - 2y = 0$. Use reduction of order to find the general solution to $t^2y'' - 2y = t^6$ with $t > 0$.