307 EXAM 2 REVIEW

(1) $ay'' + by' + cy = 0$ { 2 real roots; 1 real root; Complex roots }

(2) $ay'' + by' + cy = g(t)$ { Undetermined coeff.; Variation of Parameters }

(3) $y'' + p(t)y' + q(t)y = g(t)$ { Reduction of Order; Variation of Parameters }

(4) MASS-SPRING $\Rightarrow$ w, m, L, k, g, γ

$w - kL = 0$

$w = mg$

$F_d = -\gamma u'$

$g = 9.8$ or 32

$12\text{ in} = 1\text{ ft}$

↓ undamped

critically damped & overdamped

transient sol'n & steady state sol'n

amplitude, frequency, period } ω_0, μ, ω

quasi quasi

resonance

(5) Wronskian

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + Y(t)$$

EXAMPLES

OLD EXAM QUESTIONS

A $y'' + 3y = \sin(t) + 5e^{-t}$

$r^2 + 3 = 0 \qquad r = \pm\sqrt{3}\, i$

$y_1(t) = \cos(\sqrt{3}\, t)$
$y_2(t) = \sin(\sqrt{3}\, t)$

$Y(t) = A\cos(t) + B\sin(t) + Ce^{-t}$
$Y'(t) = -A\sin(t) + B\cos(t) - Ce^{-t}$
$Y''(t) = -A\cos(t) - B\sin(t) + Ce^{-t}$

$y'' + 2y \overset{?}{=} \sin(t) + 5e^{-t}$

$-A\cos(t) - B\sin(t) + Ce^{-t} + 3A\cos(t) + 3B\sin(t) + 3Ce^{-t} \overset{?}{=} \sin(t) + 5e^{-t}$

$2A\cos(t) + 2B\sin(t) + 4Ce^{-t} = \sin(t) + 5e^{-t}$

$2A = 0 \qquad A = 0$
$2B = 1 \qquad B = 1/2$
$4C = 5 \qquad C = 5/4$

$$y(t) = c_1\cos(\sqrt{3}t) + c_2\sin(\sqrt{3}t) + \tfrac{1}{2}\sin(t) + \tfrac{5}{4}e^{-t}$$

CHANGE 3 TO 1 $\Rightarrow$ WHAT CHANGES
CHANGE 3 TO -1 $\Rightarrow$ WHAT CHANGES
CHANGE 3 TO -1 $\Rightarrow$ WHAT CHANGES
AND CHANGE $5e^{-t}$ TO $5te^{-t}$

13 $t^2 y'' + 2ty' - 2y = 0$

$y_1(t) = t^{-2}$ FIND GENERAL SOLN

$y(t) = u(t)\, t^{-2} = u t^{-2}$

$y' = u' t^{-2} - 2u t^{-3}$

$y'' = u'' t^{-2} - 2u' t^{-3} - 2u' t^{-3} + 6u t^{-4}$

$= u'' t^{-2} - 4u' t^{-3} + 6u t^{-4}$

$t^2 y'' + 2t y' - 2y \stackrel{?}{=} 0$

$t^2(u'' t^{-2} - 4u' t^{-3} + 6u t^{-4}) + 2t(u' t^{-2} - 2u t^{-3}) - 2(u t^{-2}) = 0$

$u'' - 4t^{-1}u' + 6t^{-2}u + 2t^{-1}u' - 4t^{-2}u - 2t^{-2}u = 0$

$u'' - 2t^{-1}u' = 0$ $\quad$ $\boxed{v' - 2t^{-1}v = 0}$

$\mu(t) = e^{\int -2t^{-1}dt} = e^{-2\ln(t)} = t^{-2}$

$\frac{d}{dt}(t^{-2}u') = 0$

$t^{-2}u' = a_1$

$u'(t) = a_1 t^2$

$u(t) = \frac{a_1}{3}t^3 + a_2$

$y(t) = \left(\frac{a_1}{3}t^3 + a_2\right)t^{-2} = \frac{a_1}{3}t + a_2 t^{-2}$

$a_2 \quad\quad a_1$

$\boxed{y(t) = c_1 t^{-2} + c_2 t}$