Math 300 Assignment 5a

PROBLEMS: 6.9(c)(d), 7.5, 7.9, 7.11, 7.28(a), 7.30(a)

Also complete the supplemental problem below:

The following is a review with explanations of what is and what is not allowed when it comes to a congruence.

1. (ADD/SUBTRACT/MULTIPLY) Recall that you are allowed to add, subtract, and multiply congruences (included in multiplication is replacing the *base* of an exponential). For each problem find the smallest positive answer:

   (a) $8^{300} + 6^{100} \equiv ??? \pmod 7$.

   (b) Solve $x + 72^4 \equiv 31 \pmod 5$.

   (c) Find the ones digit (in the base 10) of 1099^6. ($i.e$ $1099^6 \equiv ??? \pmod{10}$)

2. (POWERS) Powers cannot be replaced, but you can use Fermat's Little Theorem and successive squaring to simplify. Use these fact or manipulation of the power to compute:

   (a) $3^{92} \equiv ??? \pmod{31}$ (31 is a prime)

   (b) $2^{40} \equiv ??? \pmod 6$ (6 is not a prime so Fermat's Little Theorem does not apply)

3. (CANCELLATION) In general, as you saw in HW 7.1, the cancellation of the c in the congruence $ac \equiv bc \pmod n$ is only allowed if $\gcd(n, c) = 1$.

   (a) In the following calculation a two has been canceled out of both sides of the congruence. Is this allowed? Explain why by calculating the appropriate gcd value:
   $14 \equiv 24 \pmod{10} \Rightarrow 7 \equiv 12 \pmod{10}$.

   (b) In the following calculation a five has been canceled out of both sides of the congruence. Is this allowed? Explain why by calculating the appropriate gcd value:
   $35 \equiv 15 \pmod 4 \Rightarrow 7 \equiv 3 \pmod 4$.

   (c) Let n is a natural number.

      i. Give a short proof using the facts above, that if $2x - 4 \equiv 2y - 4 \pmod{2n+1}$, then $x \equiv y \pmod{2n+1}$.

      ii. Give integers n, x and y such that $2x - 4 \equiv 2y - 4 \pmod{2n}$, but $x \not\equiv y \pmod{2n}$. (Thus, canceling the 2 is only allowed if the modulus is odd.)

4. (DIVISION/MULTIPLICATIVE INVERSES) In congruences **we cannot divide**. However, we can multiply in clever ways to get answers but only if we pay attention to the factors of the number in questions and how they relate to the factors of the modulus.

   (a) Find all the numbers that have inverses modulo 10 and tell me what their inverses are.

   (b) Using the previous part, solve the congruence $3x \equiv 124 \pmod{10}$. (multiply both sides by the inverse of 3).

   (c) Two numbers a and b are multiplied together to give the number c. If the ones digit of a is 7 and the ones digit of c is 4, what is the ones digit of b?

The problems above are DUE NEVER, but you must complete these to prepare for the final exam.

HOMEWORK NOTES/HINTS

- PROBLEM 7.5 AND 7.28: Don't use your calculator or long division to find the remainders (I am giving you these problem to practice modular arithmetic).
  For 7.28, write $535801 = 5 \cdot 10^5 + 3 \cdot 10^4 + 5 \cdot 10^3 + 8 \cdot 10^2 + 0 \cdot 10 + 1$. (the base 10 expansion)
  Then simplify each of the 10's modulo 7. Here are a few ways that the first powers of 10 could be simplified:
  $10 \equiv 3 \pmod 7$
  $10^2 \equiv 3^2 \equiv 9 \equiv 2 \pmod 7$
  $10^3 \equiv 10 \cdot 10^2 \equiv 3 \cdot 2 \equiv 6 \equiv -1 \pmod 7$
  and so forth. Then simplify each term and add up. To check for divisibility you are trying to see if the number is congruence to zero or not. Once again, this problem is merely here to give you practice with modular arithmetic, so don't use your calculator (you would be cheating yourself). If you practice this, you can get pretty quick.