Math 300 Assignment 4a

PROBLEMS: 6.8, 6.17

In addition, complete the following supplemental/review problems:

1. Give a counterexample to the following statement:
   Every onto function from $\mathbb{R}$ to $\mathbb{R}$ is one-to-one.

2. The two sums $A = \sum_{k=0}^{n} \binom{n}{k} (-1)^k$ and $B = \sum_{k=0}^{n} \binom{n}{k} w^k (1-w)^{n-k}$ each give a fixed constant
   for all choices of $n \geq 1$ and w. Give the two constant values A and B.

3. Let $2 \leq k \leq n$. Prove that $\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}$.

4. Let $n \geq 1$ (the first part below is for practice with factorials, the second part is more challenging
   and will not be on the test, but it's a useful fact that follows from part (a)).

   (a) Explain why the three expressions below are equal:
   $$\binom{2n}{n} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{1 \cdot 2 \cdot 3 \cdots n} 2^n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} 2^{2n}.$$

   (b) Prove that $2^n < \binom{2n}{n} < 2^{2n}$.

The problems above are DUE NEVER, but you are expected to work through them before Exam 2.

HOMEWORK NOTES/HINTS

- **Problem 6.8**: Use this problems to practice the Euclidean algorithm (I will introduce this at the end of class on Monday or beginning of class on Tuesday). Once you know the algorithm, this is quick (we may not get to the back substitution part until after the exam, but I will let you know).

- **Problem 6.17**: Use this as a problem to practice working with the definition of divides (this is like the problem from class where I defined $D(a, b) = $ ' the common divisors of a and b.').

- You should also try 6.1, 6.2, 6.6, 6.7, and 6.11 to check your understanding of the definitions and theorems.

- Problems 1, 2, and 3 above should be quick (they would be appropriate exam questions).

- Problem 4 above is meant to give you practice with factorials, start with the factorial formula we proved in class to compute $\binom{2n}{n}$. See if you can push the factorials around to get the result in part a. Part b follows from part a and will be a useful result for a problem from the last assignment (but is not an appropriate exam question).