Math 300 Assignment 4

PROBLEMS: 4.11, 4.24, 4.25, 4.26, 4.31, 4.34, 4.37, 4.45, 4.47, 5.6, 5.8, 5.30, 6.24

1. Prove that if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both decreasing, then $h = g \circ f$ is increasing. (this is not a typo!)
2. Using results from class give a short justification that $\mathbb{R} - \mathbb{Q}$ (the set of irrational numbers) is uncountable? Thus, in the sense of cardinality, the irrational numbers form a larger order of infinity than the rational numbers.

The problems above are DUE MONDAY, AUGUST 1st at lecture or during office hours.

HOMEWORK NOTES/HINTS

- PROBLEM 4.34: Try examples involving simple small sets such as $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Try different functions f and g between these sets (or similar small sets). Other examples can often be found, but start small.
- PROBLEM 4.45: Give me a structure proof, but it is okay if your proof is a little informal (meaning you can use phrases like "all elements are hit" or "some elements are missed"). Try the contrapositive or contradiction for each direction. For one direction, you may want to use the so-called Pigeonhole Principle that states "If m objects are being placed into n classes and $m > n$, then some class must contain two or more objects." That is, a function between finite sets A and B with $|A| > |B|$ must hit some element of B at least 2 times (by the Pigeonhole Principle). To show that the equivalence fails for infinite sets, give me an example where the theorem is not true for an infinite set.
- PROBLEM 4.47: Give formal proofs, that is give bijections f : even natural numbers $\to \mathbb{N}$ and g : odd natural numbers $\to \mathbb{N}$.
- PROBLEM 5.6: No proof required. Count the bijections from $\{1, 2\}$ to $\{1, 2\}$, then count the bijections from $\{1, 2, 3\}$ to $\{1, 2, 3\}$, etc. When you are confident you know the formula for the number of bijections from $\{1, 2, ..., n\}$ to $\{1, 2, ..., n\}$ then write it down.
- PROBLEM 6.24: Use induction on n. The phrase "3 divides $4^n - 1$ for every positive integer n" is defined by "for every positive integer n, there is an integer m such that $4^n - 1 = 3m$". Similarly, the phrase "6 divides $n^3 + 5n$ for every positive integer n" is defined as "for every positive integer n, there is an integer m such that $n^3 + 5n = 6m$".
- **REMINDERS**: Proofread your work to make sure it is organized and don't forget to answer all parts of each question. Now that we are several weeks into the quarter, I expect the proofs to be nicely written in paragraph form using complete sentences.
- If you finish the homework early or if you are looking for some extra practice try the following problems:

  CHALLENGE PROBLEMS: 3.34, 4.13, 4.48(especially challenging), 12.22

  These are not due, but I will award at most 1 point of extra credit per challenge problem correctly completed.