Math 300 Assignment 2

PROBLEMS: 1.50, 2.3, 2.4, 2.10abcd, 2.21, 2.24, 2.35, 2.38, 2.44c, 2.50b, 2.53

READ MY DIRECTIONS FOR 2.38 AND 2.44c ON THE BACK OF THIS PAGE
(Ignore the book directions).

DR. LOVELESS PROBLEMS

PROBLEM I: Using a truth table, show that $\neg(P \Rightarrow Q)$ is equivalent to $P \wedge \neg Q$.
(Aside: This is the logical equivalence that we are using when we write a proof by contradiction.)

PROBLEM II: Consider the statement: $P \Rightarrow Q$: "If Bob buys gas, then he will have no money." Now consider the negation $\neg(P \Rightarrow Q)$. A common error that students make is the following: *incorrect negation* "If Bob buys gas, then he will have some money." This is $P \Rightarrow \neg Q$, which is not the negation. This is not the negation because it still gives a true value when Bob does not buy gas. (If Bob does not buy gas, then this statement is still vacuously true). Give the correct negation of the original statement.

PROBLEM III: Find the Summer 2008 (Dr. Loveless) Exam 1 in the Exam Archive on my website (a link is on the left of the website or directly via www.math.washington.edu/∼aloveles/Math300Summer2011/examarchive.html). See how many of the problems you can do without looking at the solutions. Write up the solutions to problems 1 and 2 and hand them in with your homework (try to write them up before you look at the solutions).

PROBLEM IV: Many results in mathematics are obtained by taking a known proof and adjusting it to prove a more general or significant result. This exercise gives an example of such an instance: Recall from Assignment 1 (Dr. Loveless Problem IV) that if x and y are nonnegative and $xy \geq 1$, then $(1+x)(1+y) \geq C$ with $C = 4$. Notice that $C = 4$ is *best possible*, that is four is the largest number that will always work because equality can be obtained (by the example $x = 1$, $y = 1$ which makes the product equal to 4). By making appropriate changes to my proof of Assignment 1 Problem IV, see if you can find the C below and then see if you can find a pattern for $C(z)$ (You know that $C(1) = 4$, and $C(2) =$ 'your answer from part 1 below').

1. Mimicking your proof of Assignment 1 (supplement 1) find the largest possible number C such that if x and y are nonnegative and $xy \geq 2$, then $(1+x)(1+y) \geq C$.

2. Generalize the previous part by finding a formula $C(z)$ for the largest possible number C such that if x and y are nonnegative and $xy \geq z$, then $(1+x)(1+y) \geq C(z)$.

PROBLEM V: Complete the Exploratory Problem 1: *Where Sums and Product Meet*

The problems above are DUE FRIDAY, JULY 8 at lecture or during office hours.

HOMEWORK NOTES/HINTS

- Problems 2.3, 2.4, 2.10, 2.21, 2.24, and II should all be relatively quick quantifier/negation type questions. You might want to read my additional posting on quantifiers if you are having trouble. You should ask me questions by Friday if you are struggling with these problems.

- Problems 2.44c and I are truth table problems. These should be quick once you understand how to fill in a truth table. I just want you to give a truth table argument on both of these.

- Problems 2.50b and 2.53 are more involved subset proofs. You already know how to do subset proofs so you can start these now. Start by writing down the structure of each proof. Then, in your scratch work, write down in full detail and logic what it means to be in each set. Then see if you can explain how to get from one set of conditions to the other. These are good problems to practice your logical deduction skills.

- Problem 1.50a should read '$f(C \cap D) \subseteq f(C) \cap f(D)$' (some early printings of the book have the typo $\cup$ instead of $\cap$). We did an almost identical problem on Monday.

- Problems 2.35 and 2.38 are very important problems where you get to practice setting up 'if and only if' proofs. Be very clear and organized with the structure of your proofs. On Problem 2.35: You may assume that $a^2 = b^2$ implies $a = b$ or $a = -b$. On Problem 2.38: Prove part (a) in both directions using only the definitions of even and odd. Namely,

  n is even means $n = 2k$ for some integer k, and

  n is odd means $n = 2t + 1$ for some integer t.

  A contrapositive proof may help in one direction. Give a counterexample to part (b) and tell me if the statement is true in one direction (no proof needed for part (b)).

- Problems III and IV should be quick. In III, I am forcing you to find my exam archive which is a great resource for extra example problems. In IV, I am forcing you to look at the solutions for the last week's assignment and read my proof carefully. You should be reading all my posted proofs and solutions carefully.

- Problem V. This should be a 'fun' exercise. There is not much for you to do. I just want you to read through it, then do what is written in **bold**. I am trying to show you a bit of how mathematicians work (while also giving you some extra practice with reading proofs and working with inequalities).

- If you finish the homework early or if you are looking for some extra practice try the following problems:

  CHALLENGE PROBLEMS: 2.32 (give reasoning), 2.40, prove the Pythagorean Theorem

  These are not due, but if you hand them in with your homework, I will award you up to one extra credit point per problem (if you do this, then please put the challenge problems at the end and clearly label them).