NAME: ________________________________

Math 300 Assignment 1

The problems below are DUE WEDNESDAY, JUNE 29 at lecture or during office hours. Assignments handed in after 1:30 pm on Wednesday will be considered late and will not be accepted for any reason. You are allowed to miss one assignment without penalty to your grade. You may work together on homework, but you are expected to hand in your own work (in your own words).
Complete the problems on your own paper (except PROBLEM I from my problems, just fill it out and staple this page to the front of your homework).

TEXT PROBLEMS: 1.7, 1.13, 1.18, 1.21, 1.30a, 1.31a, 1.41d, 1.47a, 1.49ac, 2.9, 2.17, 2.36

DR. LOVELESS PROBLEMS

PROBLEM I: The standard arithmetic that you use daily can be deduced from the small set of axioms that appear in the textbook (Pages 16 and 17). For the remainder of this course you may assume all of the standard facts about arithmetic that are listed on these pages without having to reprove them. This exercise gives an example of how you would prove a basic arithmetic fact from the axioms. When you are limited to such a small set of axioms, it makes the task of proving even a small result more tedious. Here is a theorem with line-by-line proof, you just need to fill in the justification (all come from Definition 1.39) for each step: (To get you started, the first blank is 'M3: Identity')

Proposition 1.43(b): $x \cdot 0 = 0$.

Proof:

Consider $x \cdot 0 + x$.

$x \cdot 0 + x = x \cdot 0 + x \cdot 1$	____________
$x \cdot 0 + x = x(0 + 1)$	____________
$x \cdot 0 + x = x \cdot 1$	____________
$x \cdot 0 + x = x$	____________
There exists w such that $x + w = 0$	____________
Thus $(x \cdot 0 + x) + w = x + w$ gives	
$x \cdot 0 + (x + w) = x + w$	____________
So $x \cdot 0 + 0 = 0$	
Hence, $x \cdot 0 = 0$	____________

IMPORTANT PROOF NOTE: For the remainder of this course, if you use standard arithmetic (*i.e.* correctly manipulating an equation or inequality) in a proof you can justify your work by saying 'by simple arithmetic' (or a similar phrase). That is, you do not need to find the exact axiom that you are using when you are manipulating an equation, just do it. If you are doing algebraic simplifications in a proof, you should show enough steps so that the reader can follow along (show at least every other intermediate simplification if there are many steps). ALL OTHER (NON-ALGEBRAIC) DEDUCTIONS, INFERENCES, AND LEAPS IN REASONING MUST BE CLEARLY JUSTIFIED BY AN AXIOM, THEOREM, OR LOGIC (AND YOU MUST STATE THE REASON).

PROBLEM II: Identify the flaw in the argument **and** give a counterexample to the statement.

false Statement: If a is an even integer, then $\frac{a+2}{4}$ is an integer.

false proof: Let $\frac{a+2}{4} = k$ for some integer k.
Multiplying both sides by 4 gives $a + 2 = 4k$.
Solving for a gives $a = 4k - 2 = 2(2k - 1)$.
Hence a is even. (QED)

FLAW: ____

COUNTEREXAMPLE: ____

PROBLEM III: Identify the flaws in each of the following arguments.

false Statement: If $x = 0$, then $1 = 0$.

false proof 1: Assuming $1 = 0$, if we multiply both sides by x we get $x = 0$. (QED)

FLAW: ____

false proof 2: Assuming $x = 0$, if we divide both sides by x we get $\frac{x}{x} = \frac{0}{x}$ which implies $1 = 0$. (QED)

FLAW: ____

PROBLEM IV: Let x and y be nonnegative real numbers.
Prove that if $xy \geq 1$, then $(1 + x)(1 + y) \geq 4$.

HOMEWORK HINTS

- Problem 1.30a: Note that $(xu)^2 + (xv)^2 + (yu)^2 + (yv)^2 = (x^2 + y^2)(u^2 + v^2)$.
- Problem 1.41d: Ignore the directions in the problem. I want you to give a short (1-3 line) proof that $A \subseteq B$ and $B \subseteq C$ implies $A \subseteq C$, using the definition of '$\subseteq$' and starting the proof with 'Let $x \in A$'.
- Problem 2.17: You may use the additional assumption that $x \neq 0$ (so division by x is allowed). It is a good idea to start with the equation $g(x) = g(-x)$ and the formula for $g(x)$ and see if you can use algebra to end up with $f(x)f(-x) = 1$.
- If you have no idea how to start any of the homework, then don't worry. Just come visit me in office hours in the first week and we can get you going. Often only a few hints can get you pointed in the right direction.
- If you finish the homework early or if you are looking for some extra practice try the following problems:

  EXTRA CREDIT CHALLENGE PROBLEMS: 1.25, 1.31b, 1.47b

  These are not due, but if you complete and hand them in with your homework, I will award you up to one extra credit point per problem (if you do this, then please put the challenge problems at the end and clearly label them).

HOMEWORK POLICIES

- Write legibly and leave plenty of space between problems.
- In most cases, your first draft of a problem will not be sufficient. Much like other writing course you may need to write a second or third draft to get a proof clear, concise and presentable. Thus, I suggest first attempting a problem on scratch paper.
- Staple the problems together in order.
- Clearly identify your answers.
- As a reference as to length, my solutions for Assignment 1 nicely fit on five pages (but it is not uncommon for students to use several more or less pages).

HOMEWORK GUIDE

1. Most problems in this course fit into 3 categories:

   (a) PROOFS: For any homework problem that asks you to "prove" or "show" something (both words mean the same thing), you'll need to write a complete, rigorous mathematical proof, with due attention to the conventions of mathematical writing that will be explained in the course. Clearly state your reasoning, write in complete sentences and justify all your steps. I WILL BE GRADING THE PROOFS MOST CAREFULLY, SO WRITE THEM UP AS NICELY AS YOU CAN!

   (b) COUNTEREXAMPLES: For any problem that asks you to give an example to show that a statement is false, you need to clearly state your example and you need to show why it is a counterexample (that is, give an example where the hypothesis of the statement is true and the conclusion of the statement is false).

   (c) SHORT ANSWER: In addition to proofs, there are problems in the book that test your understanding of definitions and problems that test your reasoning skills. For any problem that doesn't say proof or show, you simply need to give the solution and show your work (no proof required).

For the first assignment, you should be able to tell that the problems are classified as follows:

1. PROOFS: 1.13, 1.30a, 1.31a, 1.41d, 1.47a, 1.49, 2.17
2. COUNTEREXAMPLES: 1.7, 1.49
3. SHORT ANSWER: 1.21, 2.9