Math 300 Winter 2011 Exam 1

Name: _____

Student ID Number: _____

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- USE YOUR TIME WISELY! START EACH PROBLEM BY OUTLINING THE PROOFS/IDEAS ON THE PAGE. THEN SPEND NO MORE THAN 10 MINUTES PER PAGE FILLING IN THE DETAILS (AND THEN MOVE ON TO THE NEXT PAGE).
- I will primarily be giving points for structure and format of proofs. Keep your proofs minimal, but make sure to justify each step.
- Put your answers on the exam, there are 4 pages (you may use your own paper for scratch work, but everything you want grade must be on the exam). If you need more space for your answer use the back of the preceding page and indicate that you have done so.
- If you don't know how to prove something, then you should still let me know everything you know about the problem (that is, give definitions and facts you know about the problem and discuss different techniques to prove the theorem). Also, if you are stuck you should try some examples. Don't leave any question blank, show me what you know!

GOOD LUCK!

1. (12 pts) For each statement below, give the negation AND tell me which statement is true.

   (a) There exists $a \in \mathbb{N}$ such that, for all $b \in \mathbb{N}$, if ab is even, then $a + b$ is odd.

   CIRCLE WHICH IS TRUE **TRUE**: ORIGINAL NEGATION

   (b) For all $x, y \in \mathbb{R}$, if $x + y > 10$, then $x > 0$ or $y > 0$.

   CIRCLE WHICH IS TRUE **TRUE**: ORIGINAL NEGATION

2. (14 pts) Answer the following questions about the statement: For all $n \in \mathbb{N}$, for all $q \in \mathbb{R}$, $nq \leq q^n$.

   (a) Assume we tried to give a proof by induction on n.

      i. (4 pts) Is the base step true? (Explain).

      ii. (6 pts) Properly state the regular inductive hypothesis and the strong inductive hypothesis for this statement.
      REGULAR INDUCTIVE HYPOTHESIS:

      STRONG INDUCTIVE HYPOTHESIS:

   (b) (4 pts) The statement is false. Give a counterexample.

3. (14 points) Let A, B, and C be sets. Give a properly formatted subset proof, based on definitions and logic, that

$$(A^c \cup (B - C))^c \subseteq C \cup (A - B).$$

4. (9 pts) Give a carefully organized proof based only on known facts from the fact sheet that for all real numbers a and b, $|\sqrt{|a||b|} + a + b| \leq \frac{3}{2}(|a| + |b|)$.

5. (15 pts) Define a sequence by $a_1 = -1$ and $a_{n+1} = a_n + 2n - 1$ for all $n \in \mathbb{N}$. Using a formal proof by induction, prove that $a_n = n^2 - 2n$ for all $n \in \mathbb{N}$.

6. (16 pts) Let a and b be integers.
   Clearly giving a well structure proof and using the precise definitions of even and odd:
   Prove a is even and b is even **if and only if** $a + ab + b$ is even.
   (Hint: Try indirect methods and cases for one of the directions.)