1. (14 pts)

(a) Consider $\mathbf{r}(t) = \left\langle t^2, 6t, \frac{2}{t} \right\rangle$.

Find the positive value of t at which the the acceleration vector and velocity vector are orthogonal for $\mathbf{r}(t)$ AND give the tangential component of acceleration, a_T, at this time.

$$\vec{r}\,'(t) = \langle 2t, 6, -\tfrac{2}{t^2} \rangle$$
$$\vec{r}\,''(t) = \langle 2, 0, \tfrac{4}{t^3} \rangle$$
$$\vec{r}\,' \cdot \vec{r}\,'' = 0 \Rightarrow 4t - \frac{8}{t^5} = 0$$
$$\Rightarrow 4t^6 - 8 = 0$$
$$t^6 = 2$$
$$t = \pm 2^{1/6}$$

NOTE: $a_T = \dfrac{\vec{r}\,' \cdot \vec{r}\,''}{|\vec{r}\,'|} = 0$ at this time

$t =$ $2^{1/6}$

a_T 0

(b) Find $f_x(x, y)$ and $f_y(x, y)$ for $z = f(x, y) = y^x + y\sin(x^2y) + \ln(x)$.

$f_x(x, y) =$ $y^x \ln(y) + 2xy^2\cos(x^2y) + \frac{1}{x}$

$f_y(x, y) =$ $xy^{x-1} + \sin(x^2y) + x^2y\cos(x^2y)$

(c) Find $\frac{\partial z}{\partial y}$ for $z^3 + y^2z = x\sqrt{3+y^2}$ at $(x, y, z) = (1, 1, 1)$.

$$3z^2\frac{\partial z}{\partial y} + 2yz + y^2\frac{\partial z}{\partial y} = \frac{\cancel{2}xy}{\cancel{2}\sqrt{3+y^2}}$$

at $(1,1,1)$ WE HAVE

$$3\frac{\partial z}{\partial y} + 2 + \frac{\partial z}{\partial y} = \frac{1}{2}$$
$$\Rightarrow 4\frac{\partial z}{\partial y} = -\frac{3}{2}$$

$\frac{\partial z}{\partial y} =$ $-3/8$

2. (12 pts) **The two parts below are not related.**

(a) Find and classify all the critical points for $f(x, y) = 8x\cos(y) - x^2$ such that $0 \le y \le \pi$. (*Clearly, show work using the 2nd derivative test and label your final points as local max/min or saddle points. You do NOT have to find the z-values, just the* (x, y) *points*).

① $f_x = 8\cos(y) - 2x = 0$

② $f_y = -8x\sin(y) = 0$ ONLY VALUE IN RANGE

② $\nearrow$ $x = 0 \xrightarrow{①} 8\cos(y) - 0 = 0 \rightarrow y = \pi/2$ $(0, \pi/2)$

$\searrow$ $\sin(y) = 0 \rightarrow y = 0 \xrightarrow{①} 8 - 2x = 0 \rightarrow x = 4$ $(4, 0)$

ONLY IN RANGE $\searrow$ $y = \pi \xrightarrow{①} -8 - 2x = 0 \rightarrow x = -4$ $(-4, \pi)$

$f_{xx} = -2,\ f_{yy} = -8x\cos(y),\ f_{xy} = -8\sin(y)$

$(0, \pi/2) \Rightarrow D = (-2)(0) - (-8)^2 = -64 < 0$

$(4, 0) \Rightarrow D = (-2)(-32) - (0)^2 = 64 > 0$ $f_{xx} < 0$

$(-4, \pi) \Rightarrow D = (-2)(-32) - (0)^2 = 64 > 0$ $f_{xx} < 0$ } CONCAVE DOWN

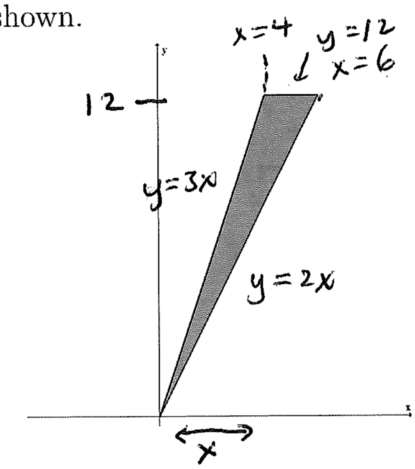
SADDLE PT LOCAL MAX

List of Critical Points: $(0, \pi/2)$ $(4, 0),\ (-4, \pi)$

(b) Consider $\int_0^{12} \int_{y/3}^{y/2} g(x, y)\, dx dy$. Rewrite the equivalent integral(s) after reversing the order of integration (Do NOT evaluate). A picture of the region is shown.

$x = y/2 \Rightarrow y = 2x \overset{?}{=} 12 \rightarrow x = 6$

$x = y/3 \Rightarrow y = 3x \overset{?}{=} 12 \rightarrow x = 4$



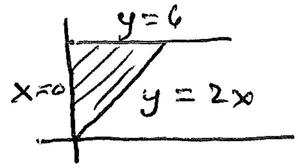
Rewritten integral(s): $\int_0^4 \int_{2x}^{3x} g(x,y)\,dy\,dx + \int_4^6 \int_{2x}^{12} g(x,y)\,dy\,dx$

3. (12 pts) **The two parts below are not related.**

(a) Find the volume of the solid in the first octant that is bounded by $y = 6$, $y = 2x$ and $z = x^2$.

$$\iint_R x^2\,dA$$

OR



$$\int_0^6 \int_0^{y/2} x^2\,dx\,dy = \int_0^3 \int_{2x}^6 x^2\,dy\,dx$$

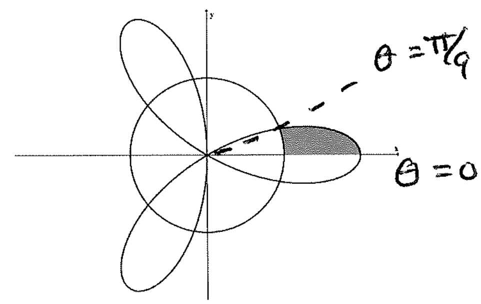
$$\int_0^6 \frac{1}{3}x^3 \Big|_0^{y/2} dy$$

$$\int_0^6 \frac{1}{24} y^3\,dy$$

$$\left[\frac{1}{24}\frac{1}{4}y^4\Big|_0^6\right] = \frac{1}{24}\frac{1}{4}6^4 = \frac{6\cdot 6\cdot 6}{4\cdot 4 \cdot 2} = \frac{27}{2}$$

Volume = $\frac{27}{2} = 13.5$

(b) Find the area of the region shown which is between $x^2 + y^2 = 1$, $r = 2\cos(3\theta)$ and above the x-axis.



$x^2+y^2=1 \Rightarrow r=1$

INTERSECTION: $1 = 2\cos(3\theta)$

$\Rightarrow \frac{1}{2} = \cos(3\theta)$

$3\theta = \pi/3 \quad \theta = \pi/9$

$$\iint_R 1\,dA = \int_0^{\pi/9}\int_1^{2\cos(3\theta)} 1\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/9} \frac{1}{2}r^2 \Big|_1^{2\cos(3\theta)} d\theta$$

$$= \int_0^{\pi/9} 2\cos^2(3\theta) - \frac{1}{2}\,d\theta = \int_0^{\pi/9} 1 + \cos(6\theta) - \frac{1}{2}\,d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{6}\sin(6\theta)\Big|_0^{\pi/9}$$

$$= \frac{\pi}{18} + \frac{1}{6}\sin\left(\frac{2\pi}{3}\right)$$

Area = $\frac{\pi}{18} + \frac{\sqrt{3}}{12}$

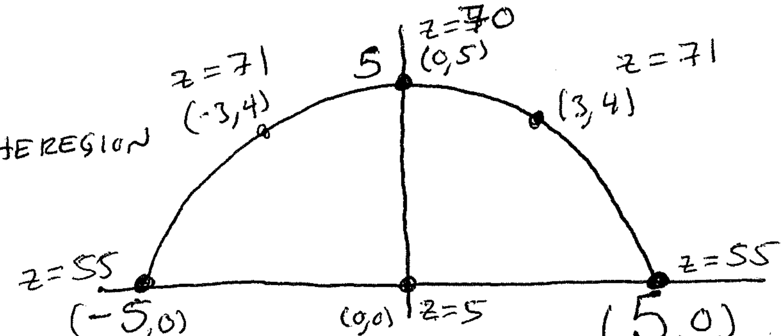
4. (12 pts) Find the absolute (*i.e.* global) minimum and maximum values of the function $z = f(x, y) = 5 + 2x^2 + y^2 + 8y$ on the region $D = \{(x, y) : y \geq 0, x^2 + y^2 \leq 25\}$.

(*For full credit: You must very clearly show that you found and analyzed the one variable functions above each boundary curve and that you checked all appropriate points and endpoints*)

$f_x = 4x \stackrel{?}{=} 0 \rightarrow x = 0$

$f_y = 2y + 8 \stackrel{?}{=} 0 \rightarrow y = -4$

$(0, -4)$ IS NOT IN THE REGION



$\boxed{\text{I}}$ $y = 0$, $-5 \leq x \leq 5 \Rightarrow z = f(x, 0) = 5 + 2x^2$

$z' = 4x = 0 \Rightarrow x = 0$

$$\boxed{\begin{array}{l} x = -5 \Rightarrow f(-5,0) = 55 \\ x = 0 \Rightarrow f(0,0) = 5 \\ x = 5 \Rightarrow f(5,0) = 55 \end{array}}$$

$\boxed{\text{II}}$ $x^2 + y^2 = 25$, $0 \leq y \leq 5$, $-5 \leq x \leq 5$ TWO WAYS

$x^2 = 25 - y^2$

$z = 5 + 2(25 - y^2) + y^2 + 8y$

$z = 55 - 2y^2 + y^2 + 8y$

$z = 55 - y^2 + 8y$

$z' = -2y + 8 = 0 \Rightarrow y = 4$

$$\boxed{\begin{array}{l} y = 0 \Rightarrow z = f(\pm 5, 0) = 55 \\ y = 4 \Rightarrow z = f(\pm 3, 4) = 55 - 16 + 32 = 71 \\ y = 5 \Rightarrow z = f(0, 5) = 70 \end{array}}$$

$y = \sqrt{25 - x^2}$

$z = 5 + 2x^2 + 25 - x^2 + 8\sqrt{25 - x^2}$

$z' = 2x + \dfrac{-8x}{\sqrt{25 - x^2}} \stackrel{?}{=} 0$

$2x\sqrt{25 - x^2} - 8x = 0$

$2x(\sqrt{25 - x^2} - 4) = 0$

$x = 0$ OR $25 - x^2 = 16$

$x^2 = 9$

$x = \pm 3$

$$\boxed{\begin{array}{l} x = -5 \rightarrow z = 55 \\ x = -3 \rightarrow z = 71 \\ x = 0 \rightarrow z = 70 \\ x = 3 \rightarrow z = 71 \\ x = 5 \rightarrow z = 55 \end{array}}$$

global max: $z =$ 71

global min: $z =$ 5