Math 126 - Spring 2023
Exam 2

Name _____

Student ID #_____

Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- This exam consists of this cover, four pages of questions, and a blank "scratch sheet". If you put work on the scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see scratch page".

- You will have 50 minutes.

- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**) and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.

- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.

- **You must show your work.** The correct answer with no supporting work may result in no credit.

- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.

- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

- There may be multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

1. (13 pts) **The two parts below are not related.**

   (a) (6 pts) Let $z = g(x, y)$ be a function defined implicitly by the equation
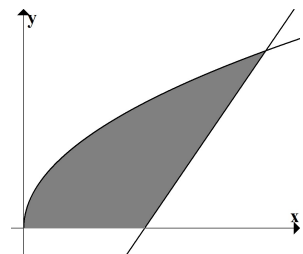 $$6z^2 = x^2 z + xy - \ln(y).$$
   Find the tangent plane to $g(x, y)$ at the point $(x, y, z) = (2, 1, 1)$. (*Hint: Find both partials*)

   tangent plane equation: _____

   (b) (7 pts) Let R be the region in the first quadrant of the xy-plane bounded by $y = x - 2$ and $y^2 = x$ (as shown). Evaluate
   $$\iint\limits_R 2y \, dA.$$

   .

   Answer = _____

2. (13 pts) **The two parts below are not related.**

(a) (9 pts) Let $z = f(x, y) = 6x^2 - 3x^2 y + y^3 - 3y$.
Find and classify all the critical points for $f(x, y)$.
(*Clearly, show work using the 2nd derivative test and label your final points as local max/min or saddle points. You do NOT have to find the z-values, just the (x, y) points*).

List of Critical Points: _____

(b) (4 pts) Rewrite the following integral after reversing the order of integration (do NOT evaluate).
(*Just draw the region, reverse the order of integration and rewrite the integral*)

$$\int_0^2 \int_{x^2}^4 x \sin(y^2) \, dy \, dx.$$

Rewritten integral: _____

3. (12 pts) **The two parts below are not related.**

(a) (4 pts) Consider the region, R, in the xy-plane that is inside the circle of radius 3 and above the x-axis. Set up the the following integral in two ways, in the order $dxdy$ and in polar. (*Do NOT evaluate, meaning give bounds and rewrite as an iterated integral, then stop!*)

$$\iint\limits_{R} \sin(x^2 + y^2)dA$$

$dxdy$ set up: _____

Polar set up: _____

(b) (8 pts) Find the area of the region in the fourth quadrant outside the Cardiod $r = 1 + \sin(\theta)$ and instead the circle $x^2 + y^2 = 4$. A picture of this region is below.
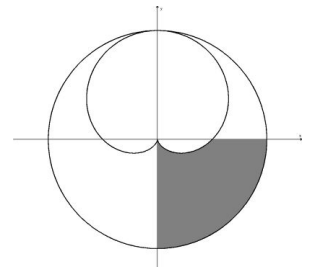
Area = _____

4. (12 pts) Your job is to design a box with a bottom, four side walls, one dividing wall in the middle and no top (as shown). All walls are made of plywood. You are told that the total volume must be 6 cubic feet and that cost of plywood is 3 dollars per square foot.

Find the minimum cost to produce the box.
(*You MUST give a two variable function for cost and find its critical point as part of your answer, you do NOT have to do the second derivative test*).
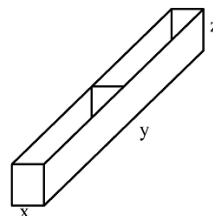
minimum Cost = _____ dollars

You may use this page for scratch-work or extra room.

All work on this page will be ignored unless you write and circle "see scratch page" on the problem and you label your work.