Math 126
Exam 1
April 20, 2023

Name _____

Student ID #_____

Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- This exam consists of this cover, four pages of questions, and a blank "scratch sheet". If you put work on the scratch sheet and you want it to be graded, then you must clearly tell us in the problem to "see scratch page".
- You will have 50 minutes.
- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**) and one 8.5 by 11 inch sheet of handwritten notes (front and back). All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam. You may not listen to headphones or earbuds during the exam.
- **You must show your work.** The correct answer with no supporting work may result in no credit.
- Leave your answer in exact form. Simplify standard trig, inverse trig, natural logarithm, and root values. Here are several examples: you should write $\sqrt{4} = 2$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\frac{7}{2} - \frac{3}{5} = \frac{29}{10}$ and $\ln(1) = 0$ and $\tan^{-1}(1) = \frac{\pi}{4}$.
- Unless otherwise indicated, when rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- There may be multiple versions, you have signed an honor statement, and cheating is a hassle for everyone involved. If we find that you give an answer that is only appropriate for the other version of the exam and there is no work to support your answer, then you will get a zero on the entire exam and your work will be submitted to the academic misconduct board. JUST DO NOT CHEAT.

GOOD LUCK!

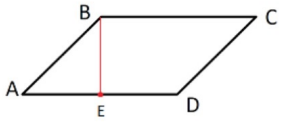
1. (12 pts)

(a) (3 pts) Consider $x^2 - 4y^2 - z = -9$.

- Give the 2D name of the traces when $x = k$ is fixed. Name: _____
- Give the precise name of the 3D shape given by $x^2 - 4y^2 - z = -9$

Name: _____

(b) Consider the parallelogram shown with $A(1, 1, 2)$, $B(2, 3, 7)$, $C(5, 3, 11)$, $D(4, 1, 6)$.



i. (4 pts) Find the area of the parallelogram ABCD.

Area = _____

ii. (3 pts) Find the vector of length 5 that points in the same direction as $\overrightarrow{BD}$.

Vector: _____

iii. (2 pts) The line segment BE (shown) is perpendicular to the segment AD. Find the length of the segment AE.

Length = $|AE|$ = _____

2. (14 pts)

(a) (2 pts) True or false, circle one for each statement:

- TRUE or FALSE : Two planes are always parallel or intersecting.
- TRUE or FALSE : Two planes perpendicular to a given plane must be parallel.

(b) (6 pts) Consider the line that contains the point $(5, 0, 0)$ and is orthogonal to the plane $3y - 4z = 10$. Find the two points of intersection of this line with the paraboloid $20x = y^2 + z^2$. (First find parametric equations for the line!)

Intersection Points: $(x, y, z) =$ _____

(c) (6 pts) Find an equation for the plane that passes through the point $(0, 0, 2)$ and contains the line of intersection of the planes $x + y - z = 1$ and $2x + y - 3z = -1$. And give the x-intercept of this new plane equation.

Plane Equation: _____

x-intercept: $(x, y, z) =$ _____

3. (12 pts)

(a) (6 pts) Find the angle of intersection of the curves: (Round final answer to the nearest degree)

- $\mathbf{r}_1(t) = \langle t, 3 - t, t^4 \rangle$
- $\mathbf{r}_2(u) = \langle 2 - u, 2u - 2, 10 - u^2 \rangle$

Angle = _____

(b) (6 pts) Let C be the curve of intersection of the surface $y = \frac{1}{2}x^2$ and the surface $z = \frac{1}{3}xy$. Parameterize this curve, then use the parameterization to give the arc length of the curve from the point $(0, 0, 0)$ to $(3, \frac{9}{2}, \frac{9}{2})$

Arc Length = _____

4. (12 pts)

(a) (2 pts) True or false, circle one for each statement:

i. TRUE or FALSE : $\mathbf{r}'(t)$ and $\mathbf{N}(t)$ are always orthogonal.

ii. TRUE or FALSE : $\mathbf{r}''(t)$ and $\mathbf{N}(t)$ are always parallel.

(b) Consider $\mathbf{r}(t) = \langle t^2, 3t + 6, -2t^2 \rangle$.

i. (5 pts) Find the curvature at $t = 0$. (Give your answer as a decimal rounded to three digits)

$\kappa(0) =$ _____

ii. (5 pts) Find the equation of the tangent line to $\mathbf{r}(t)$ at the point $(4, 12, -8)$ and find where this line intersects the xz-plane

Intersection with xy-plane: $(x, y, z) =$ _____

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see scratch page" on the problem and you label your work.