1. (13 pts) **The two parts below are not related.**

(a) (6 pts) Let $z = g(x, y)$ be a function defined implicitly by the equation

$$6z^2 = x^2 z + xy - \ln(y)$$

Find the tangent plane at the point $(x, y, z) = (2, 1, 1)$.

$12z\frac{\partial z}{\partial x} = 2xz + x^2\frac{\partial z}{\partial x} + y$ @ $(2,1,1)$

$12\frac{\partial z}{\partial x} = 4 + 4\frac{\partial z}{\partial x} + 1 \Rightarrow \frac{\partial z}{\partial x} = \frac{5}{8}$

$12z\frac{\partial z}{\partial y} = x^2\frac{\partial z}{\partial y} + x - \frac{1}{y}$ @ $(2,1,1)$

$12\frac{\partial z}{\partial y} = 4\frac{\partial z}{\partial y} + 2 - 1 \Rightarrow \frac{\partial z}{\partial y} = \frac{1}{8}$

tangent plane equation: $\boxed{z - 1 = \frac{5}{8}(x-2) + \frac{1}{8}(y-1)}$

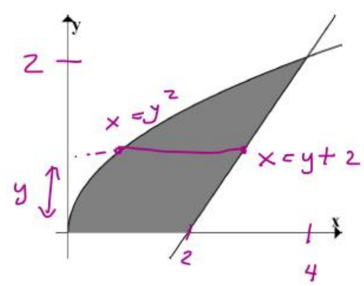
(b) (7 pts) Let R be the region in the first quadrant of the xy-plane bounded by $y = x - 2$ and $y^2 = x$ (as shown). Evaluate

$$\iint_R 2y\,dA$$

$y = y^2 - 2$

$\Rightarrow 0 = y^2 - y - 2 = (y-2)(y+1)$

$y = 2$ or $y = -1$



$\int_0^2 \int_{y^2}^{y+2} 2y\,dx\,dy$

$\int_0^2 2yx\Big|_{y^2}^{y+2} dy = \int_0^2 2y(y+2) - 2y^3\,dy$

$= \int_0^2 2y^2 + 4y - 2y^3\,dy$

$= \frac{2}{3}y^3 + 2y^2 - \frac{1}{2}y^4\Big|_0^2$

$= \frac{16}{3} + \cancel{8} - \cancel{8} = \boxed{\frac{16}{3}}$

OTHER WAY:

$\int_0^2 \int_0^{\sqrt{x}} 2y\,dy\,dx + \int_2^4 \int_{x-2}^{\sqrt{x}} 2y\,dy\,dx$

Answer = $\boxed{16/3}$

2. (13 pts) **The two parts below are not related.**

(a) (9 pts) Let $z = f(x, y) = 6x^2 - 3x^2y + y^3 - 3y$.
Find and classify all the critical points .
(*Clearly, show work using the 2nd derivative test and label your final points as local max/min or saddle points. You do NOT have to find the z-values, just the points*).

$f_x = 12x - 6xy \overset{?}{=} 0 \rightarrow 6x(2-y) = 0 \Rightarrow x = 0,\ y = 2$

$f_y = -3x^2 + 3y^2 - 3 \overset{?}{=} 0$

$x = 0 \Rightarrow 3y^2 - 3 = 0 \Rightarrow y = \pm 1$

$y = 2 \Rightarrow -3x^2 + 12 - 3 = 0 \Rightarrow 3x^2 = 9 \Rightarrow x^2 = 3$
$\Rightarrow x = \pm\sqrt{3}$

$f_{xx} = 12 - 6y$

$f_{yy} = 6y$

$f_{xy} = -6x$

$(0,1) \Rightarrow D = (6)(6) - (0)^2 = 12 > 0$ MIN

$(0,-1) \Rightarrow D = (18)(-6) - (0)^2 < 0$ SADDLE

$(-\sqrt{3}, 2) \Rightarrow D = (0)(12) - (6\sqrt{3})^2 < 0$ SADDLE

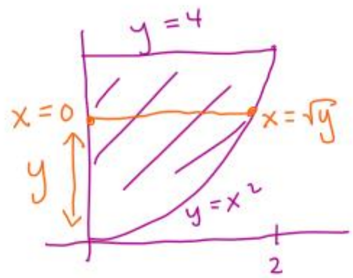
$(\sqrt{3}, 2) \Rightarrow D = (0)(12) - (6\sqrt{3})^2 < 0$ SADDLE

MIN   SADDLE POINTS

List of Critical Points: $(0,1)$ $(0,-1)$, $(-\sqrt{3}, 2)$ $(\sqrt{3}, 2)$

(b) (4 pts) Rewrite the following integral after reversing the order of integration. (*Do NOT evaluate, just reverse the order of integration and rewrite the integral*)

$$\int_0^2 \int_{x^2}^4 x \sin(y^2)\, dy dx.$$



Rewritten integral: $\int_0^4 \int_0^{\sqrt{y}} x \sin(y^2)\, dx\, dy$

3. (12 pts) **The two parts below are not related.**

(a) (4 pts) Consider the region, R, in the xy-plane that is inside the circle of radius 3 and above the x-axis. Set up the the following integral in two ways, in the order $dxdy$ and in polar. (*Do NOT evaluate, meaning give bounds and rewrite as an interated integral*)

$x^2+y^2=9$

$\Rightarrow x = \pm\sqrt{9-y^2}$

$$\iint_R \sin(x^2+y^2)dA$$

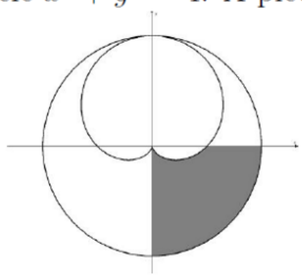
$dxdy$ set up: $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \sin(x^2+y^2)\,dx\,dy$

Polar set up: $\int_0^{\pi} \int_0^3 \sin(r^2)\, r\,dr\,d\theta$

(b) (8 pts) Find the area of the region in the fourth quadrant that is outside the Cardiod $r = 1+\sin(\theta)$ and instead the circle $x^2+y^2=4$. A picture of this region is below.

$-\frac{\pi}{2} \le \theta \le 0 \quad \left(\text{OR } \frac{3\pi}{2} \le \theta \le 2\pi\right)$

$1+\sin\theta \le r \le 2$



$$\iint_R 1\,dA = \int_{-\pi/2}^{0} \int_{1+\sin\theta}^{2} r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{0} \frac{1}{2}r^2 \Big|_{1+\sin\theta}^{2} d\theta$$

$$= \int_{-\pi/2}^{0} 2 - \frac{1}{2}(1+2\sin\theta+\sin^2\theta)\,d\theta$$

$$= \int_{-\pi/2}^{0} 2 - \frac{1}{2} - \sin\theta - \frac{1}{2}\cdot\frac{1}{2}(1-\cos(2\theta))\,d\theta$$

$$= \int_{-\pi/2}^{0} \frac{3}{2} - \sin\theta - \frac{1}{4} + \frac{1}{4}\cos(2\theta)\,d\theta$$

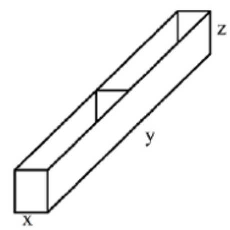
$$= \frac{5}{4}\theta + \cos\theta + \frac{1}{8}\sin(2\theta) \Big|_{-\pi/2}^{0}$$

$$= (0+1+0) - \left(-\frac{5\pi}{8} + 0 + 0\right) = \boxed{1 + 5\pi/8}$$

Area = $1 + \frac{5\pi}{8}$

4. (12 pts) Your job is to design a box with a bottom, four side walls, one dividing wall in the middle and no top (as shown). All walls are made of plywood. You are told that the total volume must be 6 cubic feet and that cost of plywood is 3 dollars per square foot.

Find the minimum cost to produce the box.
(*You MUST give a two variable function for cost and find its critical point, you do NOT have to do the second derivative test*).



$\text{COST} = 3 \cdot \text{AREA}$

$= 3 \cdot (xy + 2yz + 3xz)$

CONSTRAINT: $xyz = 6 \Rightarrow z = \frac{6}{xy}$

$\text{COST} = f(x,y) = 3xy + 6y\frac{6}{xy} + 9x\frac{6}{xy}$

$f(x,y) = 3xy + \frac{36}{x} + \frac{54}{y}$

$f_x = 3y - \frac{36}{x^2} \stackrel{?}{=} 0 \Rightarrow y = \frac{12}{x^2}$

$f_y = 3x - \frac{54}{y^2} \stackrel{?}{=} 0 \rightarrow 3xy^2 - 54 = 0$

$\rightarrow xy^2 - 18 = 0$

$x\left(\frac{12}{x^2}\right)^2 - 18 = 0$

$\frac{144}{x^3} - 18 = 0$

$x^3 = \frac{144}{18} = 8$

$\boxed{x = 2} \Rightarrow \boxed{y = \frac{12}{(2)^2} = \frac{12}{4} = 3} \Rightarrow z = \frac{6}{xy} = 1$

MIN COST $= 3 \cdot (6 + 6 + 6) = 3 \cdot 18 = \boxed{\$54}$

minimum Cost = 54 dollars