Ch. 14 - Slopes on Surfaces - Quick Summary

$z = f(a, b)$ = height above xy-plane at (a, b)	Be able to find and graph the domain. Know the basics on level curves/contour maps
$f_x(a, b) = \frac{\partial z}{\partial x}$ = slope in x-direction at (a, b) $f_y(x, y) = \frac{\partial z}{\partial y}$ = slope in y-direction at (a, b)	$\langle 1, 0, f_x(a, b)\rangle$ is a tangent vector in x-direction $\langle 0, 1, f_y(a, b)\rangle$ is a tangent vector in y-direction
$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$	Tangent plane or Linear Approximation at (a, b)
$f_{xx}(a, b) = \frac{\partial^2 z}{\partial x^2}$ = concavity in x-direction at (a, b) $f_{yy}(a, b) = \frac{\partial^2 z}{\partial y^2}$ = concavity in y-direction at (a, b)	$f_{xy}(a, b) = \frac{\partial^2 z}{\partial y \partial x}$ = mixed second partial at (a, b) $f_{xy} = f_{yx}$ (Clairaut's Theorem)
$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$ = measure of concavity	at a critical point (a, b)... $D < 0$ means concavity changes (saddle point) $D > 0$, $f_{xx} < 0$ means concave down all dir. (local max) $D > 0$, $f_{xx} > 0$ means concave up all dir. (local min)

- **To find critical points**: Find $f_x(x, y)$ and $f_y(x, y)$. Set them BOTH equal to zero, then COMBINE the equations and solve for x and y. Check that your points do in fact make both partials equal to zero!

- **To classify critical points**: First find the critical points, then find f_{xx}, f_{yy}, and f_{xy}. At each critical point compute f_{xx}, f_{yy}, f_{xy} and D. Make appropriate conclusions from the second derivative test.

- **To find absolute max/min over a region**: Draw the region.
  1. Find the critical pts inside the region, label in region.
  2. Over each boundary:
     (a) Substitute the xy-equation for that boundary into the surface to get a one variable function.
     (b) Find the critical numbers and endpoints for this one variable problem (Calculus 1). The picture of the region will tell you the endpoints.
     (c) Label all these critical numbers and endpoints on this boundary. Repeat for other boundary curves.
  3. Plug each point you found into the original function $z = f(x, y)$.
     Biggest output is the max, smallest output is the min.

- **To find a max/min in an applied problem (optimization)**
  1. Draw and label a picture.
  2. *Objective*: What are you optimizing?!? (This is the function you need to find and differentiate!)
  3. *Constraint*: Write down the constraints and use them to make the objective a two-variable function.
  4. Find the critical points of your objective function. (If you differentiate anything other than your objective, you made a mistake).
  5. If asked, use the second derivative test to verify that your critical point does indeed give a max or min as desired.

BIG SKILLS TO PRACTICE:

- Computing partial derivatives. Be able to handle all the situations you saw in homework. It is vital on this exam that you can do derivatives, even if they are 'messy'.
- Solving systems of equations to find critical points. Be organized in your work. Be logical in how you combine equations (like I showed in lecture and in the live-stream). And always, always, always check that your final points do indeed make BOTH your 1st partials zero.