1. (13 pts)

(a) Find a vector that has length 7 and is orthogonal to both $\mathbf{u} = \langle 1, 0, 2 \rangle$ and $\mathbf{v} = \langle 3, -2, 1 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 3 & -2 & 1 \end{vmatrix} = (0 - -4)\vec{i} - (1-6)\vec{j} + (-2-0)\vec{k} = \langle 4, 5, -2 \rangle$$

CHECK: $4 + 0 - 2 = 0$ ✓

$12 - 10 - 2 = 0$ ✓

$$|\vec{u} \times \vec{v}| = \sqrt{16+25+4} = \sqrt{45} = 3\sqrt{5}$$

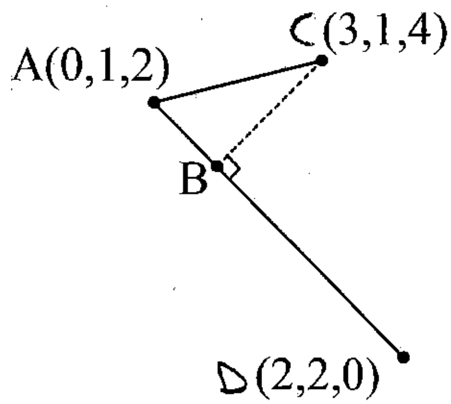
$$\boxed{\frac{7}{\sqrt{45}} \langle 4, 5, -2 \rangle} \quad \text{OR} \quad \boxed{\frac{-7}{\sqrt{45}} \langle 4, 5, -2 \rangle}$$

(b) Find the distance from point A to point B in the picture below (Hint: Use vector tools!)

$\vec{AC} = \langle 3, 0, 2 \rangle$

$\vec{AD} = \langle 2, 1, -2 \rangle$

$$\text{comp}_{\vec{AD}} \vec{AC} = \frac{6+0-4}{\sqrt{4+1+4}} = \boxed{\frac{2}{3}}$$




(c) Consider the line through the points $(0, 0, 1)$ and $(3, 4, 5)$. Find the $(x, y, z)$ point(s) where the line intersects the cylinder $x^2 + y^2 = 4$.

LINE: $x = 0 + 3t, \; y = 0 + 4t, \; z = 1 + 4t$

INTERSECTION: $(3t)^2 + (4t)^2 = 4 \Rightarrow 25t^2 = 4 \Rightarrow t^2 = \frac{4}{25}$

$\Rightarrow t = \pm \frac{2}{5}$

$t = -\frac{2}{5} \Rightarrow \boxed{(x,y,z) = \left(-\frac{6}{5}, -\frac{8}{5}, 1 - \frac{8}{5}\right) = \left(-\frac{6}{5}, -\frac{8}{5}, -\frac{3}{5}\right)}$

$t = \frac{2}{5} \Rightarrow \boxed{(x,y,z) = \left(\frac{6}{5}, \frac{8}{5}, 1 + \frac{8}{5}\right) = \left(\frac{6}{5}, \frac{8}{5}, \frac{13}{5}\right)}$

Exam I Answers
Math 126 C Winter 2018

Version 1: In #1(b), you are asked to find vectors with magnitude 4.

1. (a) (1 points each) from top to bottom: S N S V S V

1. (11 points)

(a) (5 pts) Consider the line through the points $P(1,3,-2)$ and $Q(3,5,7)$. Find the $(x,y,z)$ coordinates of the point at which this line intersects the $xz$-plane.

DIRECTION: $\vec{PQ} = \langle 2,2,9 \rangle$

LINE: $x = 1+2t$
$y = 3+2t$
$z = -2+9t$

ASIDE: There are infinitely many parametrizations of the line. But the direction must be parallel to $\langle 2,2,9 \rangle$

INTERSECT XZ-PLANE: $y=0 \Rightarrow 0 = 3+2t \Rightarrow t = -3/2$

$$\boxed{\begin{aligned}(x,y,z) &= \left(1+2\left(-\tfrac{3}{2}\right),\ 0,\ -2+9\left(-\tfrac{3}{2}\right)\right)\\ &= \left(-2,\ 0,\ -\tfrac{31}{2}\right)\end{aligned}}$$

← Everyone should get the same answer here

(b) Consider the **plane**, $P$, **that contains the point** $(1,-1,2)$ and is the orthogonal to the line given by

$$L: \begin{cases} x = -3t \\ y = 2+7t \\ z = 5-t \end{cases}$$

i. (4 pts) Find the equation for the plane, $P$.

$\langle -3,7,-1 \rangle$ is normal to the desired plane.

$\langle 1,-1,2 \rangle$ is a position vector.

So
$$\boxed{\begin{aligned}&\langle -3,7,-1 \rangle \cdot \langle x-1, y+1, z-2 \rangle = 0\\ &-3(x-1) + 7(y+1) - (z-2) = 0\\ &-3x + 3 + 7y + 7 - z + 2 = 0\\ &-3x + 7y - z + 12 = 0\end{aligned}}$$

ii. (2 pts) At what point $(x,y,z)$ does this plane intersect the $x$-axis?

$x$-axis $\Leftrightarrow$ $y=0$ and $z=0$

So $-3x + 7(0) - (0) + 12 = 0 \Rightarrow x = 4$

$$\boxed{(4,0,0)}$$