1. (11 points)

(b) Find the center and radius of the sphere with points $P(x, y, z)$ such that the distance from P to $A(0, 0, 2)$ is triple the distance from P to $B(0, 0, 0)$.

$$\sqrt{x^2+y^2+(z-2)^2} = 3\sqrt{x^2+y^2+z^2}$$

$$x^2+y^2+z^2-4z+4 = 9x^2+9y^2+9z^2$$

$$4 = 8x^2+8y^2+8z^2+4z$$

$$\frac{1}{16}+\frac{1}{2} = x^2+y^2+z^2+\frac{1}{2}z+\frac{1}{16}$$

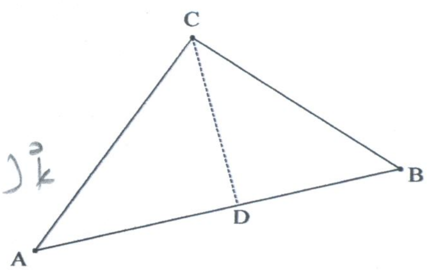
$$\hookrightarrow \frac{1}{4}$$

$$\frac{9}{16} = x^2+y^2+(z+\tfrac{1}{4})^2$$

$$\boxed{\text{CENTER} = (0,0,-\tfrac{1}{4}) \qquad \text{RADIUS} = \tfrac{3}{4}}$$

2. (14 pts) Consider the triangle shown. The coordinates for C are (4,3,1).
You are given $\overrightarrow{BA} = \langle -2, -1, 3 \rangle$ and $\overrightarrow{BC} = \langle -1, 1, 4 \rangle$.
The dotted line CD is perpendicular to BA.
Answer the following questions (Leave your answer in exact form, you do not have to simplify).

(a) (5 pts) Find the equation of the plane that contains the points A, B, and C.



$$\vec{n} = \overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 3 \\ -1 & 1 & 4 \end{vmatrix}$$

$$= (-4 - 3)\vec{i} - (-8 - -3)\vec{j} + (-2 - 1)\vec{k}$$

$$= \langle -7, 5, -3 \rangle$$

$$\boxed{-7(x-4) + 5(y-3) - 3(z-1) = 0}$$

(b) (3 pts) Find the area of the triangle ABC.

$$\text{AREA} = \frac{1}{2} |\overrightarrow{BA} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{(-7)^2 + (5)^2 + (3)^2} = \frac{1}{2}\sqrt{49+25+9}$$

$$= \frac{1}{2}\sqrt{83}$$

(c) (3 pts) Find the coordinates for the point A.

$\overrightarrow{BC} = \langle -1, 1, 4 \rangle$ and $C = (4,3,1) \Rightarrow B = (4--1, 3-1, 1-4)$

$B = (5, 2, -3)$

$\overrightarrow{BA} = \langle -2, -1, 3 \rangle$ and $B = (5,2,-3) \Rightarrow A = (5+-2, 2+-1, -3+3)$

$$\boxed{A = (3, 1, 0)}$$

(d) (3 pts) Find the distance from B to D.

$$|BD| = \text{comp}_{\overrightarrow{BA}} \overrightarrow{BC} = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{|\overrightarrow{BA}|} = \frac{(-1)(-2) + (1)(-1) + (4)(3)}{\sqrt{(-2)^2 + (-1)^2 + (3)^2}}$$

$$= \frac{2 - 1 + 12}{\sqrt{14}} = \boxed{\frac{13}{\sqrt{14}}}$$